OPTIMAL CAPACITORS PLACEMENT AND SIZING USING ANT COLONY SYSTEM

UBICACION Y DIMENSIONAMIENTO UTILIZANDO EL SISTEMA DE COLONIA DE HOMIGAS

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Abstract: the installation of shunt capacitors in electrical distribution networks can effectively reduce energy and peak power losses, while improving quality of service particularly promoting a better voltage profile. Economic and operational benefits depend mainly on the number, location and sizes of the capacitors installed. In this problem, multiple and conflicting evaluation aspects are generally at stake, such as minimizing capacitor installation cost and reducing system losses. In this paper, a metaheuristic called Ant Colony System is presented to solve the problem of capacitor placement and sizing in radial distribution networks. Sensitivity analysis was used to select the candidate locations for placing the capacitor in the distribution feeders. The estimation of these candidate locations basically helps reduce the search space for the optimization procedure. To demonstrate the effectiveness of the proposed method, two distribution systems of specialized literature were proven with rather encouraging results.

Keywords: Capacitor placement, Ant Colony System, distribution feeders, losses reduction, reactive compensation, pheromone.

1. INTRODUCTION

The ant colony optimization is a recent metaheuristic presented by Dorigo in 1990, it is designed to solve combinatorial problems. This technique was inspired from the natural behavior of the ant colonies on how they find the food source and bring them back to their nest by building the unique trail formation. It has been applied to well-known problems of specialized literature demonstrating to be a trustworthy alternative to obtain results of good quality.

This article introduces an ant colony search algorithm to solve the optimal capacitor placement problem. This algorithm solving hard combinational optimization problems.

It is a population-based approach that uses exploration of positive feedback as well as greedy search. Therefore, through a collection of cooperative agents called ants, the near-optimal solution to the capacitor placement problem can be effectively achieved. In addition, in the algorithm, the state transition rule, local pheromone-updating rule, and global pheromone-updating rule are all added to facilitate the computation. Through operating the population of agents simultaneously, the process stagnation can be effectively prevented. Namely the optimization capability can thus be significantly enhanced. Moreover, the capacitor placement problem is a combinatorial optimization problem having an objective function composed of power losses and capacitor installation costs subject to bus voltage constraints.

The problem consist in determining the points and the capacity at which a condenser bank must be installed so as to bring an economic benefit that justifies the initial investment in the purchase and installation of the banks. It is necessary to consider the load variable behaviour, the system technical restrictions and the commercial size of fixed and variable banks. The number of variables present in the system creates a space with possible solutions quite ample, which it grows exponentially as the number of nodes of the system grows.

2. PROBLEM FORMULATION

The main goal for optimal capacitor placement and sizing, in a particular distribution system, is to minimize the active power losses costs and to conserve an equipment suitable operation in the network, guaranteeing that the tension levels are conserved within specific limits: Vmin<Vi<Vmax.

It must take in account that, depending of load level connected to the network the system losses degree can be greater or smaller. In Figure 1 a discreet curve of load is shown, where Si represents the load for a time period Ti. Therefore, the value of demand for each level of load can be represented of the form $Q^i = Q^0 \cdot S_i$, Where Q^0 it represents the value from peak load demand.

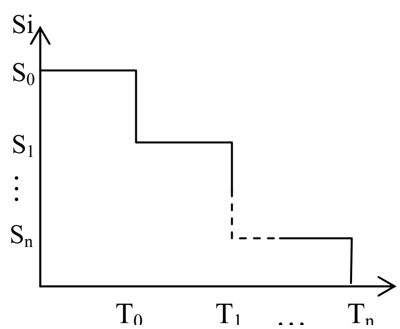

Fig. 1: Curve of demand

A power flow in each curve period Ti determines the nodal tensions value V_j and at the same time it allows to determine the active power losses P_j. In this way the energy losses cost are calculated using equation (1), where ke is a parameter that assigns a cost to the losses of energy ($/kwh.).

$$C_{energy} = k_e \cdot \sum_{j=1}^{nc} T_j \cdot P_j \qquad (1)$$

The compensation can be made with fixed banks or with variable capacity. Both are composed of fixed standard capacity units which connected to other units of similar characteristics constitute the capacity of the bank to install. By disadvantages of technical and economic type, in each compensated node there is a limited *nb* of units that can fit up. The total cost of all the banks installed in the system is then expressed in the form:

$$C_{capacitors} = \sum_{k=1}^{u} f(n_i) \qquad (2)$$

Where n_i (*with* $i=1,2..u$) it is the number of units installed in each bar where a bank installation was assigned. The parameter u is the total number of banks installed in the distribution network. From information above, the global formulation of the problem is described as:

$$\min f(s) = k_e \cdot \sum_{i=1}^{nc} T_i \cdot P_i + \sum_{k=1}^{u} f(n_i) \qquad (3)$$

Subject to:

$G_i(x_i,u_i) = 0$	*Circuitals laws*
$V_{min} < V_i < V_{max}$	*For each node*
$0 \le n_i \le n_b$	*For each bank*

During the search process of best configuration is necessary to apply repeatedly a power flow. Because of this is convenient to have an algorithm for power flow equipped for radial topologies, so that the computational load is reduced of algorithm search. In the present work it is used a back/forward sweep method (Granada, 2004).

3. ANTS COLONY SYSTEM

The ant natural behaviours and the indirect communication between all colony members to search food is the inspiration source to create an able algorithm to find good quality solutions to combinatorial problems (Maniezzo, 1996).

Denuebourg presented in 1990 an experiment made with real ants in which an ant's nest was connected to a food source by means of a bridge with two different paths length. The first ants that leave the nest find two ways without signs of pheromones. Therefore they move on equal proportion and at the same speed through both ways (Kochenberger, 2003).

The ants that walk through the path of smaller length arrive at the food source in shorter time and when they are returning the path is marked with pheromones to recognize the route. Afterwards, all the ants that arrive at the food source prefer to return to the nest by the short way since it has more intense signs of pheromones.

For this reason the process is qualified like autocatalytic or positive feedback, because the pheromone trail is increasing more quickly through the time. Nevertheless, a little ants group choose the long path with little pheromone amount. This guarantees paths exploration in trails without pheromones or with a little accumulation of this.

If ants problem is extended to several food sources connected completely among them by means of ways of different length, and it is considered that there has to be a transition by each one of the sources without going twice through same point, leads to a known combinatorial optimization problem known in the specialized literature as Hamiltonian circuit. From practical point of view, is the same problem faced by a mailman whose work is to give a series of packages among a given number of cities and designating the way of smaller possible distance.

The ant's colony systems are classified within the approximate algorithms group of constructive type for the search of good solutions to combinatorial problems. This algorithm constructs a solution iteratively, adding in each step a point (or city) j, from a group of possible alternatives Nik which lodges the places set l not visited, each one described by a probability pij of being chosen given by:

$$pij = \frac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{l \in N^k_i} \tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}} \tag{4}$$

Where τ_{ij} is the element stored in the pheromones matrix and it defines the degree of acceptance by part of previous ants to cross the way from i towards j. η_{ij} is the information coming from some heuristic technique, which at this point is defined as:

$$\eta_{ij} = \frac{1}{d_{ij}} \tag{5}$$

Where d_{ij} is the path length that connects the point i with the point j. The initial parameters α and β give the degree of importance to the information used. If α > β greater importance is given to the result information from the pheromones matrix, on the other way heuristic information prevails.

The selection process is made by means of random or deterministic mechanisms according to a random variable q_0, which is compared with an specified parameter q, both between the interval [0,1], as shown in the equation (6).

$$j = \begin{cases} \arg\max l \in N^k_i \{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}\} & si\, q \le q0 \\ J & otherwise \end{cases} \tag{6}$$

In the first case ($q \le q_0$) the selection is deterministic type, meaning that the element j is added in which there is a greater combination value of the heuristic information with the pheromones matrix. In the opposite case a random selection or stochastic mechanism is applied using probability given by the equation (4).

Once finished the construction of solutions set that conforms the ants population (where each ant represents an alternative solution) each one is evaluates within objective function (7) with the purpose of determining the construction quality:

$$f(s) = \sum_{s} d_{ij} \tag{7}$$

$i=1,2,\ldots,n$ and $j=1,2,\ldots,n$ with $i \neq j$, where n is the number of cities considered in the problem to form the alternative solution s. Once the value is known, a value inversely proportional to the solution quality is stored in the pheromones matrix, given by the following equation:

$$\Delta\tau_{ij} = \frac{1}{f(s)} \tag{8}$$

Where $\Delta\tau_{ij}$ it is the increase on the element τ_{ij} of the pheromone's matrix whose way was used in the construction of the alternative solution s. The ant colony systems only allows the pheromones deposit to the best so far alternative solution obtained throughout the search process, it is say whit the best quality evaluation of the objective function, well-known like incumbent. Optionally it is allowed to make deposits due to the best alternative solution found during an iteration, with the purpose of turning the search a little more exploratory increasing a greater number of elements of pheromones matrix, to avoid the anticipated accumulation on a specific zone of solution space that possibly leads to low quality solutions.

Like in the natural surroundings, value stored in the pheromones matrix is susceptible to evaporate and its aim is to avoid the stagnation on ways of low quality. In ant colony system evaporation process takes place in two different stages. The first stage, known as local evaporation is made during the construction of each solution, applying equation (9) to the element recently added to alternative in construction. The second stage, known as global evaporation it is executed after making the pheromone deposits pertaining to incumbent or to the best alternative solution found during the current iteration, applying in the elements that conform one or another alternative the equation (10). In both cases, ε and ρ are parameters of calibration for the algorithm in the interval [0,1].

$$\tau_{ij} = \tau_{ij} \cdot (1-\varepsilon) \tag{9}$$

$$\tau_{ij} = \tau_{ij} \cdot (1-\rho) \tag{10}$$

4. APPLICATION OF ALGORITHM TO THE PLACE CAPACITORS PROBLEM

Next the developed algorithm is described. It is based in the optimization by ant's colony for locate and size the capacitors in a three-phase balanced distribution system. It is not taken into count the effect of the harmonics.

4.1 Sensitivity analysis

It is a heuristic mechanism that allows reducing the alternative solutions space, determining a list of nodes that produce the greater impact in the active power losses when a capacitors bank is installed to them. A simple form to find the nodes most sensible is calculating the partial derived from the active power losses before to an injection of reagents (Baran 1989). The losses of the power system are expressed like:

$$P_L = \sum_{j=1}^{n}\sum_{i=1}^{n}\left|V_i V_j Y_{ij}\right|\cos(\theta_{ij}+\delta_j-\delta_i) \tag{11}$$

The level of tension in the shipment node it are expressed like $V_i e^{j\delta_i}$ and in the receipt node like $V_j e^{j\delta_j}$, connected through the line with parameters of $Y_{ij} e^{j\theta ij}$. Soon:

$$\left[\frac{\partial P_L}{\partial Q}\right] = \left[J1\right]\cdot\left[\frac{\partial P_L}{\partial V}\right] + \left[J2\right]\cdot\left[\frac{\partial P_L}{\partial \delta}\right] \tag{12}$$

Where, [*J1*] and [*J2*] are the inverse transposed of the Jacobean submatrices [*J22*] and [*J21*], respectively, and:

$$\left[\frac{\partial P_L}{\partial Q}\right] = \left[J1\right]\cdot\left[\frac{\partial P_L}{\partial V}\right] + \left[J2\right]\cdot\left[\frac{\partial P_L}{\partial \delta}\right] \tag{13}$$

$$\left[\frac{\partial P_L}{\partial \delta}\right] = -2\cdot\sum_{j=1}^{n} V_i V_j G_{ij} Sen(\delta j - \delta i) \tag{14}$$

4.2 Problem codification

An alternative solution is made up grouping *nf* of fixed banks plus an amount *nv* of variable banks. The capacity installed for each one, no matter its operation state, is determined by the number of installed units *Ni*, which have a limit *nb* $(0 \le Ni \le nb)$, where only the variable banks have the possibility of varying the number of units connected in each level of load. In this way an solution alternative is an matrix of dimensions $(nc+1)$x$(nf+nv)$, where *nc* is the load levels number within the study period.

Table 1 General configuration of one alternative solution

$$\begin{bmatrix} Node_1 & \cdots & Node_{nf} & Node_{(nf+1)} & \cdots & Node_{(nf+nv)} \\ N_1 & \vdots & N_{nf} & N(nf+1)_1 & \cdots & N(nf+nv)_1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ N_1 & \cdots & N_{nf} & N(nf+1)_{Snc} & \cdots & N(nf+nv)_{Snc} \end{bmatrix}$$

The first row corresponds to the number that identifies the compensated bars, whereas the number of units installed in each level of load for each bank is shown from row two to the row $(nc+1)$.

The pheromone deposits are accumulated on two types of matrices. The first matrix, identified like locations matrix $[\tau]_u$, of dimensions 2xn (where n is system nodes number), and in this one is expressed the desirable learning to install in bar i (with $i=1,2..n$) a fixed bank if it is in first row, or a variable bank if it is in second row.

$$\left[\tau\right]_u = \begin{bmatrix} \tau_{11} & \tau_{12} & \cdots & \tau_{1n} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2n} \end{bmatrix}_u \tag{15}$$

$$\left[\tau\right]_{c^i} = \begin{bmatrix} \tau_{1,1} & \tau_{1,2} & \cdots & \tau_{1,n} \\ \tau_{21} & \tau_{11} & \cdots & \tau_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{(nb+1),1} & \tau_{(nb+1),2} & \cdots & \tau_{(nb+1),n} \end{bmatrix}_{c^i} \tag{16}$$

The second matrix, identified like capacities matrix for the load level i, with dimensions $(nb+1)$xn, expresses in first row the desirable learning for to install no one capacitor's units in bar k (*with* $k=1,2..n$) during the load level i (*with* $i=1,2..nc$). Row two indicates preference to locate a single unit; row three, to install two units, and successively until the row $(nb+1)$ in order to install nb units. If it is desired to install variable capacitors $(nb>0)$ is required to form one matrix of this type for each load level.

The mechanism to construct solutions is developed in two phases. First stage determines which nodes are more attractiveness for fixed bank installation and which nodes for variable banks installation. For every node there is a probability of being chosen given by the equation (18). The first ants population only uses the most attractive nodes heuristically (α=0 and β=1), whereas the following constructions make use exclusively the information contained in the locations matrix (α=1 and β=0). If it is desired to install a fixed bank, then it will consider the elements on first row (*j=1*) from the locations matrix (16). Whereas, if is located a variable bank to considers the elements into second row from the same matrix

$$p_{kj} = \frac{\tau_{kj}{}^{\alpha} \cdot \eta_{kj}{}^{\beta}}{\sum_{l \in N^k{}_i} \tau_{kl}{}^{\alpha} \cdot \eta_{kl}{}^{\beta}} \tag{17}$$

The second construction phase consists in determining the banks capacity installed in each *k* node selected from the first stage. Thus apply into each load level the equation (18) on elements into k column of corresponding capacities. In particular case that *nf*>0 and *nv*=0 (only fixed capacitors) just one capacities matrix is used. If nf>0, *nv*>0 and is required to determine one fixed bank capacity, it must be applied equation (20) to consider thus the fixed capacity effect in all load levels.

$$p_{n,k^i} = \frac{\tau_{n,k^i}}{\sum_{j=1}^{nc+1} \tau_{j,k^i}} \tag{18}$$

$$p_{n,k^i} = \frac{\tau_{n,k^1} + \tau_{n,k^2} + \ldots + \tau_{n,k^{nc}}}{\sum_{j=1}^{nc+1} \tau_{j,k^1} + \ldots \sum_{j=1}^{nc+1} \tau_{j,k^{nc}}} \tag{19}$$

The search algorithm improves remarkably its performance with addition of local search technique, in which there are made small variations to each constructed alternative by each iteration. A local change is considered to increase or decrease in one step the number of units installed in each fixed bank. With variable banks, the change only applies on one load levels.

Each local move is evaluated within objective function to determine the quality of movement. If the function is better quality then is maintained the change, otherwise the configuration back to the original state. Both cases produce a change in the value stored in the corresponding capacities matrix. The rule is given by following equation:

$$\tau_{ij} = \tau_{ij} \cdot \frac{f(s)}{f(s')} \tag{20}$$

If local move produces a lower cost, then the relation of functions is greater than one, accumulating more pheromone trail on the element that produce the change. In the other hand, if the relation is smaller than one, the corresponding element decreases his pheromone trail.

The rule for the increase of pheromones is applied at the end of each iteration to *m* best solution found throughout all the search process, diminishing its intensity as the incumbent moves away:

$$\Delta\tau = \frac{1}{w \cdot f(s_w)} \tag{21}$$

Where *w* is the classification rank (*w= 1,2,..m*) and *f(sw)* is objective function evaluation (5) on *w-th* best alternative. The evaporation is applied in local form and global form by equations (9) and (10) respectively

5. TEST RESULTS

The proposed approach is demonstrated employing two application examples. The test systems are the same as those used in Grainger (1982) and Gallego (2001).

5.1 9-bus systems

The objective function has the form:

$$\min f(s) = k_e \sum_{i=1}^{nc} T_i P_i + K_o P_o + K_c \sum_{j=1}^{uc} C_j \tag{22}$$

Where C_j is the total installed capacity using 300 *Kvar* units. The parameters: k_e=0.06 \$/kwh, K_0=168 \$/kw and K_c=4.9 \$/kvar. P_o corresponds to power losses in peak load. Data for demand curve are: *S1*= 1.1, *S2*= 0.6, *S3*= 0.3, *T1*= 1000, *T2*= 6760, *T3*= 1000 hours. Objective function evaluation without compensation for one year of study is equivalent to \$329.041. For fixed banks location case was considered four fixed banks, with an upper bound of four units per bank, the algorithm found like best one configuration, with a value in the objective function of \$308.286. This configuration was:

Table 2: Location of fixed banks (kVAr)

Node	1	4	5	6
Capacitor (kVar)	300	300	600	1200

For the installation of variable banks are assumed the same values in parameters and restrictions. The result obtained as good as the result show in Gallego (2001), where it consider the case of five variable banks and objective function of $ 306.920. The configuration obtained with an objective function evaluation of $307.092 was:

Table 3: Location of variable banks (kVAr)

Nodes

Load level	1	2	5	6
S1	300	300	900	1200
S2	300	0	600	1200
S3	0	300	0	600

However, in each case equal value to install fixed and variable banks is considered. This is not true in the real life, because the variable banks require adding equipment more expensive. For this reason was proved growing 10% to value of variable banks installation. Thus, the configuration showing in Table 2 grow value to $308.442, been upper to install four fixed banks. Next was proved installing two fixed banks and two variables banks, in which the following configuration was obtained.

Table 4: Location of variable and fixed banks (kVAr)

Nodes

Load level	1	2	5	6
S1	300	300	900	1200
S2	0	300	900	1200
S3	0	0	900	1200

The objective function has value of $307.622 in this case, which is lower than install four variable banks.

5.2 69-bus system

The data of this system are inside Baran (1989). The valuation of costs in capacitors is composed of fixed value $1000 by each bank plus a value of $900 by each unit installed in the bank. Parameter Ke=0.06$/kwh and Ko=0. For operation without compensation was found an objective function value of $146.400. For the compensation with fixed banks the best solution found corresponds to the following configuration:

Table 5. Location of fixed banks (kVAr)

Node	16	50
KVar	300	1200

Where objective function has value of $105.688. In addition an alternative solution superior by only $ 1 was obtained, in which the bank of 300 KVar was located in node 17. For installation of variable banks are assumed the same values in parameters and restrictions. In this time the configuration found takes the form as:

Table 6: Location of variables banks (kVAr)

Nodes

Load level	1	2	5	6
S1	300	300	900	1200
S2	0	300	900	1200
S3	0	0	900	1200

Thus the objective function value is $104.305, with an investment cost of $12.100. However, just like in work presented in Gallego (2001), if the number of allowed units that can be installed in every bank is increasing to five, the objective function can be improved to value $ 103.512. To this case the corresponding configuration is

Table 7: Location of variable banks (kVAr)

Nodes

Load-level	17	50
S1	600	1800
S2	300	1200
S3	300	600

6. CONCLUSIONS

The solution alternatives found in this work agree to a large extent with the data found in Gallego (2001), with exception of the case of the location of two fixed condensers and two variable condensers in the system of 9 nodes, since it is a case for which results are not revealed. The results presented make believe that a precise calibration between the parameters of exploration and exploitation will allow reaching good solution.

This is a hard, large scale combinatorial problem in which the number of local minimum solution points and the number of options to be analyzed increases exponentially with the size of the distribution system.

Sensitivity analysis has been used in the literature to reduce the set of candidate buses for capacitor addition. In this case, sensitivity measures the impact of a change in the reactive injection in a given bus on the active power losses on the distribution system.

In spite of this it is possible to affirm that ants colony optimization is a good alternative to obtain answers of good quality applied to combinatorial optimization problems into distribution systems.

REFERENCES

Gallego, R.A., Monticelli, A.J., Romero, R. (2001) *Optimal capacitor placement in radial distribution networks*. Power Systems, IEEE Transactions on Volume 16, Issue 4, Page(s):630 – 637

Granada, M., Garcés, A. and Gallego, R.A. (2004) *Three-phase flow of load for Systems Radial of Distribution*. Magazine Scientia ET Technica, No 27. Universidad Tecnológica de Pereira UTP. Colombia

Dorigo, M. and Stüttzle, T. (2004) *Ant Colony Optimization*. Library of Congress Cataloging-In-Publication Data. Massachusetts Institute of Technology. London, England.

Maniezzo Vittorio Doringo Marco And Colorni Alberto. (1996) *The ant system: Optimization by a colony of cooperating agents*. IEEE Trans. on Systems, Man, and Cybernetics – Part B,, 26(1):1–13.

Colorni, A. Dorigo M. Ma_Oli F. Maniezzo V. Righini G. And Trubian M. (1994). *Heuristics from nature for hard combinatorial optimization problems*. International Transactions in Operational Research, 3(1), 1-21.

Kochenberger, Gary A, and Glover Fred. (2003) *Handbook of metaheuristics*. Kluwer academic publishers, 1 edition.

Baran, M.E. and Wu, F.F., (1989) *Optimal Sizing of Capacitors Placed on to Radial Distribution System*. IEEE Transaction on Power Delivery. Vol 4. Not 1. pp 735-743.

Wu, F.F. and Baran, M.E., (1989) *Reconfiguration network in for Distribution Systems Loss Reduction and Load Balancing*. IEEE Transactions on Power Delivery. Vol 4 Not 2. pp 1401-1407.

Sundhararajan, S. and Pahwa To., (1994) *Capacitors Optimal Selection for Radial Distribution systems Using a Genetic Algorithm*. IEEE Transactions on Power Systems. Vol 9. Not 3. pp 1499-1505.

Huang, C., Yang, H.T. and Huang C.L. (1996) *Solving the Capacitor Placement Problem in a Radial distribution System Using Tabu Search Approach*. IEEE Transactions on Power Systems. Vol 11. Not 4. pp 1868-1873.

Grainge J. Jr. and Lee S. H. (1982) *Capacity release by shunt capacitor placement on distribution feeders: A new voltage-dependent model*. IEEE Trans. Power Apparatus and Systems, vol. PAS-101, no. 15, pp. 1236–1244