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A COMPUTER MODEL FOR POROUS FLOW IN UNDERGROUND SYSTEMS

UN MODELO COMPUTACIONAL PARA FLUJO POROSO EN SYSTEMAS SUBTERRÁNEOS

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Abstract: In this paper a 3-D computer model for the quantitative analysis of underground flow is presented. This generic model can be applied to simulate underground flows which, mainly are characterised by flow through porous media or combined free/porous regimes in sub-surface regions. This paper covers the extension of this model to combined flow systems as well as flow through unsaturated domains where the advancing flow front needs to be tracked. In order to make the software accessible for the novice user, the incorporation of the present model into a previously developed computer network system is described.

Resumen: En este artículo se presenta un modelo computacional en tercera dimensión para el análisis cuantitativo de flujos subterráneos. Este modelo genérico puede ser aplicado para simular flujos subterráneos, los cuales se caracterizan principalmente por que pasan a través de medios porosos o regímenes combinados libre/poroso en regiones bajo la superficie. Este trabajo cubre la extensión de este modelo a sistemas de flujo combinado, así como también para flujos a través de dominios no saturados, donde es necesario hacer un seguimiento al flujo circulante. Para hacer este software accesible a usuarios novatos, se describe la incorporación del modelo propuesto en un sistema de red de computadores previamente desarrollado.

Keywords: Porous flow, Combined flow, Heterogeneous porous media, Finite volume method, Computer implementation.

1. INTRODUCTION

Mathematical analysis of underground flow of viscous fluids has been the subject of extensive research in the past decades. This has resulted in the formulation of a range of equations which describe mass and momentum balance for these regimes.

The main criterion for deciding which one of available mathematical models is best suited for a particular situation is the permeability of the porous region in which the flow occurs.

In cases where the porous domain consists of densely packed particles the small pores between solid sections allow a fluid to seep through whilst the solid matrix carries all of the stresses in the system.

The most appropriate equation of motion under these circumstances is based on the well known Darcy model which relates pressure drop for flow through a porous region to the flow rate (flux) and physical parameters such as domain permeability and fluid viscosity. In contrast, if the permeability of a porous region is high enough to let the porous regime to be, at least, partially dominated by viscous forces the Brinkman model which includes viscous stresses in the momentum balance for the fluid flow is used. It can be said that in the Brinkman model the fluid/solid system is effectively regarded as a pseudo-fluid Combined free/porous flow regimes are found in groundwater hydrology, glaciology, conjunctive surface and subsurface flow, mud cake formation during drilling of oil wells, and others (e.g. see, Cieszko and Kubik, 1999 and, Das and Nassehi, 2003). Free flow in large openings under the ground can be laminar or turbulent and a judgment based on conditions of any given problem must be made. However, in this work it has been only considered situations where the free regime remains laminar and modelled it using the well known Navier-Stokes equations.

In the present work, a previously developed technique for combining free and porous regimes along arbitrary shape heterogeneous interfaces (Das, 2001; Das, 2002; Das et al., 2002; Das and Nassehi, 2003) has been extended to model industrially important underground flow systems. To maintain the generality of the analysis, the domain of interest is defined to have unbounded/open/flow-through ends.

The propagation of a flow front under the ground, which is a characteristic of unsaturated flow has also been included in the present model. Here a method based on a modified version of the Volume of Fluid (VOF) technique is used (Nassehi, 2002).

There are also cases such as flow through unsaturated domains where the advancing flow front needs to be tracked is discussed. An effective method for tracking of the flow front is therefore presented. As can be expected the resulting software is a sophisticated program whose effective application requires many hours of training and experience. In order to make the software accessible for the general user, who may only have limited experience in characterised by an assumed ‘effective viscosity’. In addition to the described models, for intermediate situations a combination of these two models have also been used (Bear, 1972; Nield and Bejan, 1992).

By its nature the Darcy model represents creeping (i.e. near zero Reynolds number) flow and cannot take into account inertia effects and no-slip wall conditions. On the other hand the determination of effective viscosity in the Brinkman model is not straightforward and can cause significant uncertainty in the accuracy of the model predictions. Both models have been used to model combined free/porous flows. Development of effective combined models is usually complicated and specific techniques need to be employed (Das and Nassehi, 2003, Nassehi, 1998). Examples of mathematical modelling, the incorporation of the present model into a previously developed computer network system is described. This system has been structured to include an intelligent front-end for complicated hydro-environmental flow modelling and provides on-line assistance for non-expert users at every stage of an application, computer implementation of the entire scheme is via a user-friendly network for multi-user applications. Therefore it significantly facilitates the utilization of the present model.

2. MATHEMATICAL MODEL

The continuity equation representing conservation of matter in all types of flow regimes simulated by the present model is based on the following mass balance equation for incompressible fluids

$$\nabla \cdot \mathbf{V} = 0 \qquad (1).$$

Where $\mathbf{V}$ is the velocity vector. However, depending on the nature of the flow, i.e. whether it is free or porous and permeability is lower than a given threshold or not a suitable equation of motion representing the conservation of momentum should be used. The following general form has been considered

$$A(\mathbf{V}) + \nabla p = 0 \qquad (2).$$

Where p is the pressure. For $A(\mathbf{V}) = \rho \dfrac{\partial \mathbf{V}}{\partial t} + \mathbf{I} \cdot \mathbf{V}$, where ρ is fluid density and $\mathbf{I}$ is the identity matrix multiplied by fluid viscosity (η) over domain permeability (k), equation (2) represents the Darcy model. For $A(\mathbf{V}) = \rho \dfrac{\partial \mathbf{V}}{\partial t} + \mathbf{I} \cdot \mathbf{V} - \mu_e \nabla^2 \mathbf{V}$, where μ_e is the 'effective' viscosity, equation (2) provides the momentum balance relation corresponding to the Brinkman model. In these models when the porous domain is considered to be heterogeneous viscosity/permeability ratio should be written as a row vector with η / k_{xx} and η / k_{yy} as its components and multiplied by the identity matrix.

Finally for $A(\mathbf{V}) = \rho \dfrac{\partial \mathbf{V}}{\partial t} + \rho \mathbf{V} \nabla \cdot \mathbf{V} - \nabla \eta \nabla \cdot \mathbf{V}$, where η is viscosity of a generalised Newtonian fluid, equation (2) yields the Navier-Stokes equation for laminar free flow in the absence of body forces. For a purely Newtonian fluid, where viscosity is constant μ, the stress term in the operator expression for the free flow can simply be written as $-\mu \nabla^2 \mathbf{V}$. In modelling steady state systems the temporal derivative term in the above definitions vanish. Combining free flow and Brinkman models does not present any particular mathematical problems and these regimes can simply be simulated conjunctively provided that effective viscosity in the porous region is matched with the fluid viscosity at the interface. However, lack of a second order velocity derivative in the Darcy equation precludes straightforward linking of free and porous equations for this case. A very well established technique for such linking is the use of interface conditions suggested by (Beavers and Joseph 1967). Details of the use of this technique for underground flow systems has been reported by (Das and Nassehi 2003). Momentum transfer between the porous and free flow domains in this method is manipulated by assuming the existence of a slip layer at the interface separating the regions. The method has also been extended to combined flows in heterogeneous underground regions (Das *et a*l, 2003). This method has been used in the present work. The advancing flow front is simulated using the VOF (volume of fluid) technique. This method is based on the solution of the front position probability density equation, given as

$$\frac{\partial F}{\partial t} + \mathbf{V}.\nabla F = 0 \qquad (3).$$

Where $0 \le F \le 1$ is called the flow front position function. In this work a modified version of the VOF technique has been used in which the moving front flow is considered as a two-phase regime in which the wet and the dry sections represent the phases (Thompson, 1986). The flow model is solved for the entire domain whilst at each section physical properties relevant to that phase are inserted. A set of fictitious physical values that render the velocity in the dry regions as zero are used and hence the technique is called the pseudo-density method. Simultaneous solution of equation (3) with the flow model generates values of *F*. A value of *F* between 0 and 1 (usually $F = 0.5$) is taken as the boundary between phases representing the moving front. Values of physical parameters for each phase in the flow field is related to the position of the front using:

$$y = y_w F + y_d (1 - F) \qquad (4).$$

Where y is a given physical parameter and y_f and y_a are the values of this parameter in the wet and dry regions, respectively, (Nassehi and Ghoreishy, 1997).

The described governing equations are discretised and reduced to algebraic forms using the standard finite volume method (FVM) (Patankar, 1980; Versteeg and Malalasekera, 1995). This method is adopted due to its inherent capability in coping with complex 3-D domain types.

The scheme calculates the flow variables on a 3-D forward-staggered grid where cell faces are ahead of cell nodes, figure1. The computational grid for both free and porous regions involves cubicle cells for storing the flow variables. While the velocity components are evaluated at different cell faces, pressure terms are predicted at cell nodes such that they act as the driving potential for flow at intermediate cell faces.

Detailed descriptions of the mesh and schemes used can be found in (Das and Nassehi 2001).

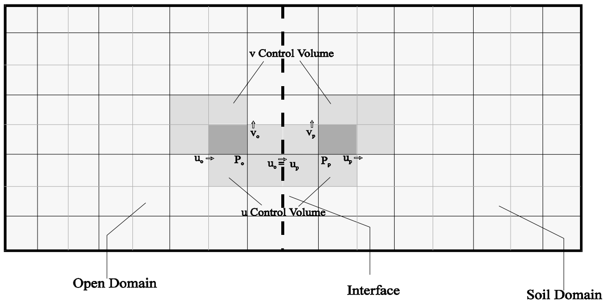

Fig. 1. Schematic representation of finite volume cell used for computation.

3. COMPUTER IMPLEMENTATION

To achieve the goals of flexibility, practicality, speed, multi-user capability and computing economy the use of the described model is based on its implementation via an Information Processing Tool.

The detail of the basic system has been previously published (N.Keshavarzi-Roonizi et al, 2004). This system has extended in this work and developed a new architecture in order to make it suitable for porous and combined free/porous problems. The IT tool consists of three modules shown in figurer 2 and can be briefly defined as:

- Front-end, provides a network node for communication between the users and the software. Quantitative results generated by the model are treated in a back-end module and the final result is provided in clearly tabulated or graphical formats. A log is kept by the system which records the particulars of the application for future comparisons and evaluation of the efficiency of the solution generated for a given problem. The front end will guide the user towards inserting appropriate input data. The system compares given permeability of the porous domain and selects the most suitable mathematical model to carry out computations. The front end facilitates data handling whilst monitoring the suitability and relevance of the data inserted by the users to activate the data generator module.

- Data-generator is a systematic library of various modelling software written in FORTRAN and symbolic programming as well as genetic algorithms,(Passone, 2002; Mokhtarzadeh, 1998; Das, 2001) to solve the model equations. The front-end interface feeds the input data to the modelling sections initiating numerical computations. At the end of the computations the simulation results are passed to the back-end module for further processing required before presentation of the answers.

- Back-end, provides a network node for the import and processing of the data (i.e. results, answers, evaluations) generated by the data generator module. The processed results are logged and returned to a window in the front-end for the utilization by the users. It should be noted that the simulations results obtained via the data-generator mainly consist of large amounts of numerical data and hence the back–end module re-organizes them as pre-formatted tables or graphical out put. The back-end module can also communicate directly with the front-end at the preliminary stages of an application (i.e. problem simulation).

For example, it is designed to provide answers regarding the validity of a given input on the basis of its self-consistency or logical status. This feature is mainly utilized to guide the untrained user to obtain required information from the system with ease and efficiency. In figure 3 the general architecture and a basic procedure of the present IT system is shown.

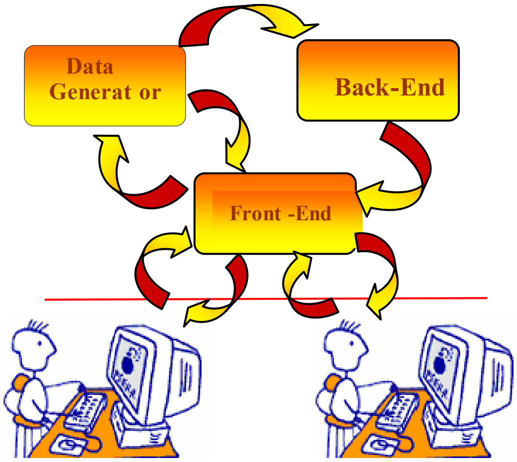

Fig. 2. Schematic of the IPT System

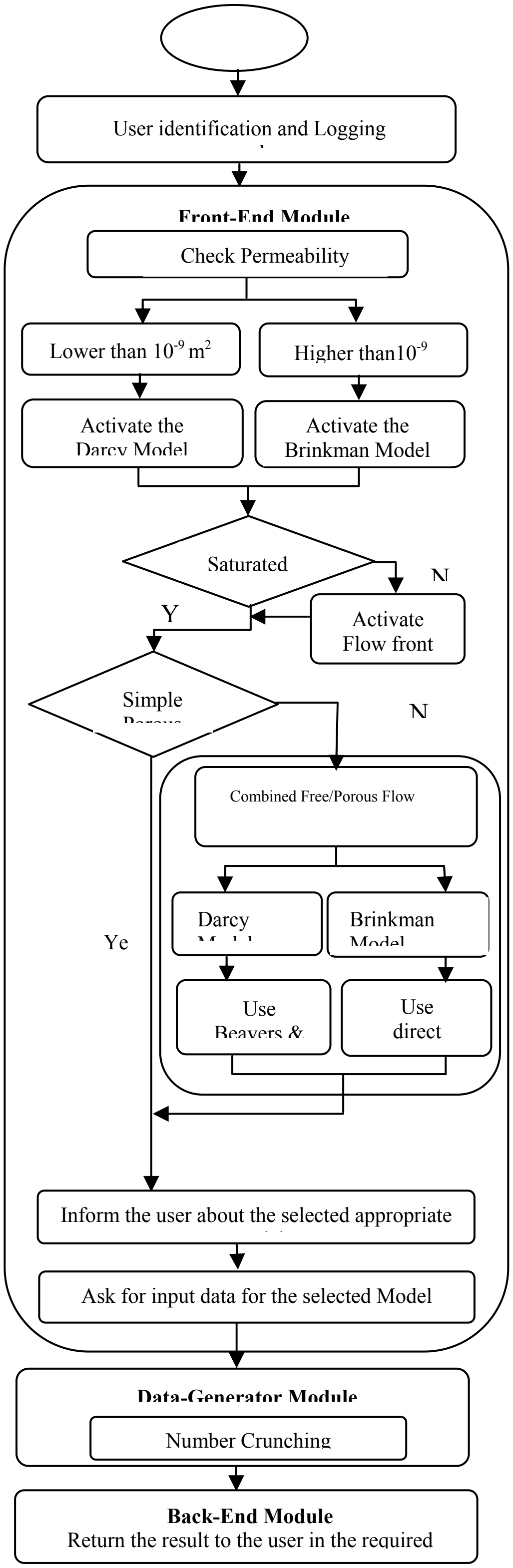

Fig. 3. Schematic diagram of the IT system

4. SAMPLE RESULT

To demonstrate the application of the present model a simulation of a combined free/porous flow regime through a domain has been considered as shown in figure 4.

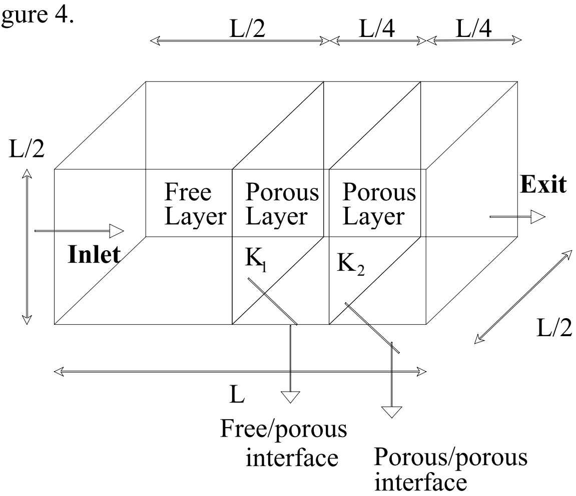

Fig. 4. Representative 3-D configurations for free flow and layered porous medium. The direction of the layer is perpendicular to the longitudinal (x-) velocity component

In order to clearly show the simulated velocity field in the following figures only the velocity vectors on the interfaces between free and porous and two porous layers of different permeability are shown. The permeability of both porous regions is taken to be lower than selected threshold and hence porous flow in these simulations is modelled by the Darcy equation. Therefore slip flow at the interface between the free and porous regions are used. However, porous/porous interface is handled normally and only different values of the permeability for these regions have been used. The front-end in the present system guides the user to insert appropriate input at every stage.

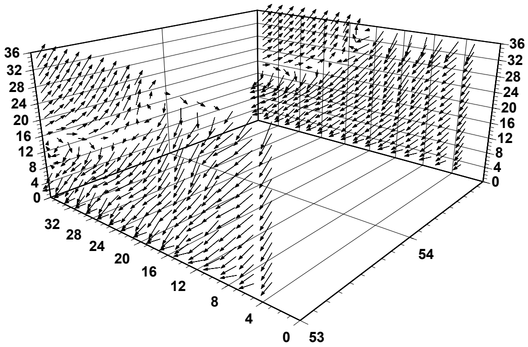

Fig. 5. Simulated Velocity Field at Porous/Porous Interface, 19th Time Level ($K_1 = 5 \times 10^{-9}$ m^2, $K_2 =$

15×10^{-11} m^2)

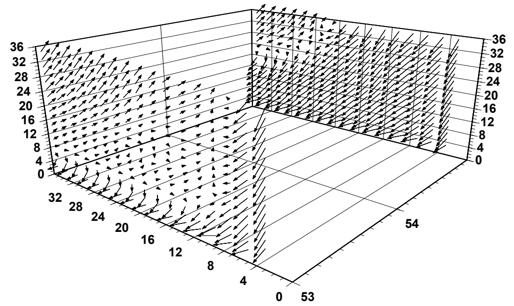

Fig. 6. Simulated Velocity Field at Porous/Porous Interface, 19th Time Level ($K_1 = 15 \times 10^{-11}$ m^2, $K_2 = 5 \times 10^{-9}$ m^2)

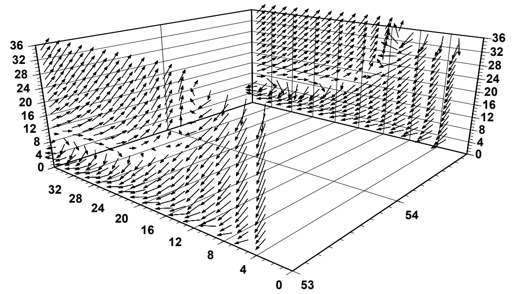

Fig. 7. Simulated Velocity Field at Porous/Porous Interface, 37th Time Level ($K_1 = 5 \times 10^{-9}$ m^2, $K_2 = 15 \times 10^{-11}$m^2)

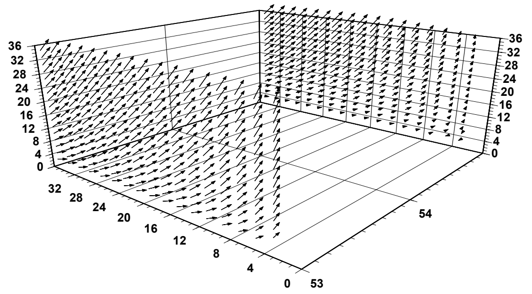

Fig. 8. Simulated Velocity Field at Porous/Porous Interface, 37th Time Level ($K_1 = 15 \times 10^{-11}$ m^2, $K_2 = 5 \times 10^{-9}$ m^2)

As the sample results show the flow pattern at the interface is quite complicated and complex interactions between the regions can be observed. This type of non-uniform flow is expected to occur under the ground as shown by information related to the study of leachates appearing from under ground regions.

5. CONCLUSIONS

The developed computer implementation system has been used to study a complex model for the effects of heterogeneity on flow hydrodynamics in coupled free/porous regimes in subsurface.

The obtained results are consistent with the sequential physical phenomena. The system has provided significant simplification and flexibility for the general user and as such should be considered as a novel hydroinformatics tool.

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