CLASS

Second Unit

PRESSURE

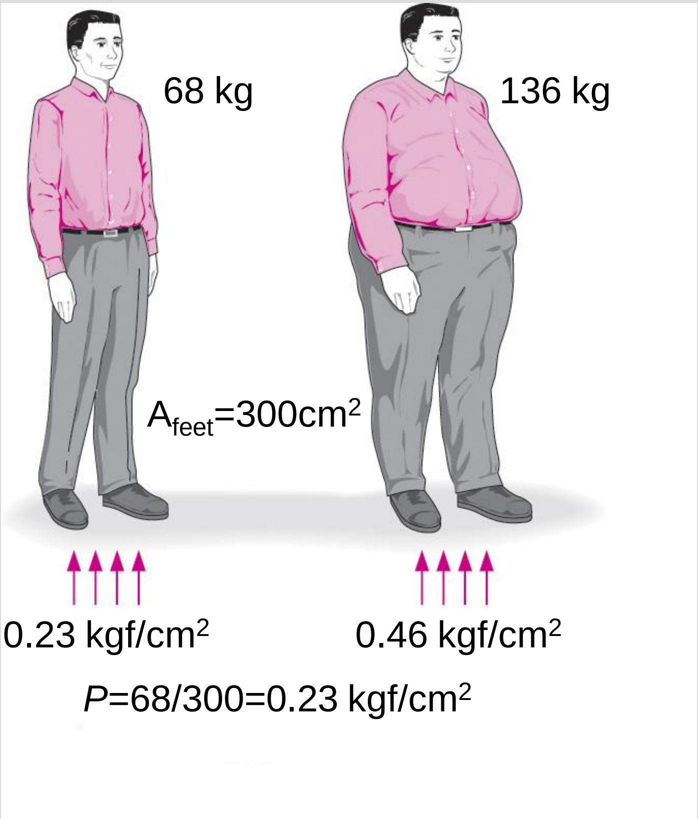
Pressure: A normal force exerted by a fluid per unit area

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$\begin{aligned} 1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} \\ &= 0.9807 \text{ bar} \\ &= 0.9679 \text{ atm} \end{aligned}$$



The normal stress (or “pressure”) on the feet of a chubby person is much greater than on the feet of a slim person.

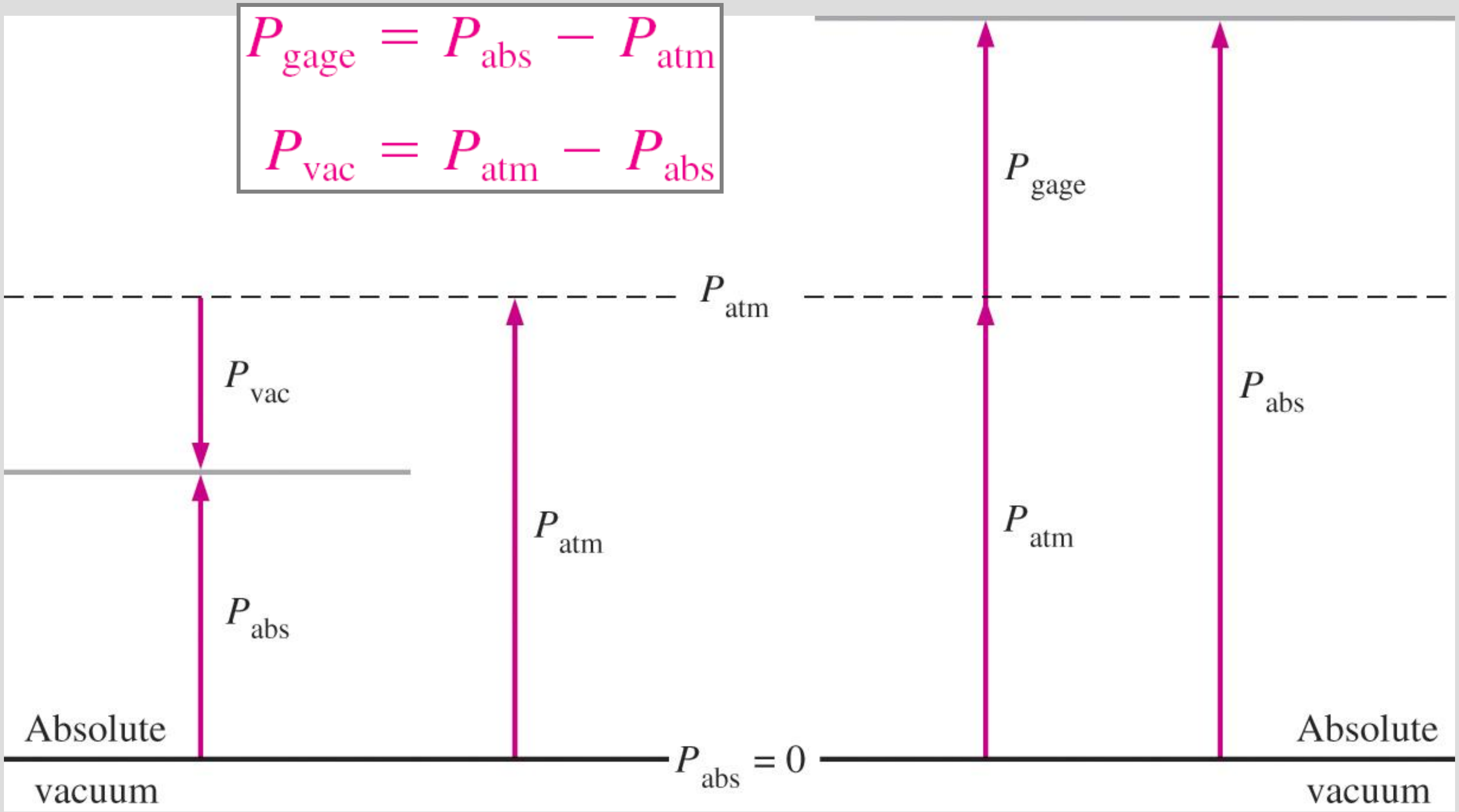
Some basic pressure gages.

- **Absolute pressure**: The actual pressure at a given position. It is measured relative to absolute vacuum (i.e., absolute zero pressure).
- **Gage pressure**: The difference between the absolute pressure and the local atmospheric pressure. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and so they indicate gage pressure.
- **Vacuum pressures**: Pressures below atmospheric pressure.

Throughout this text, the pressure ***P*** will denote ***absolute pressure*** unless specified otherwise.

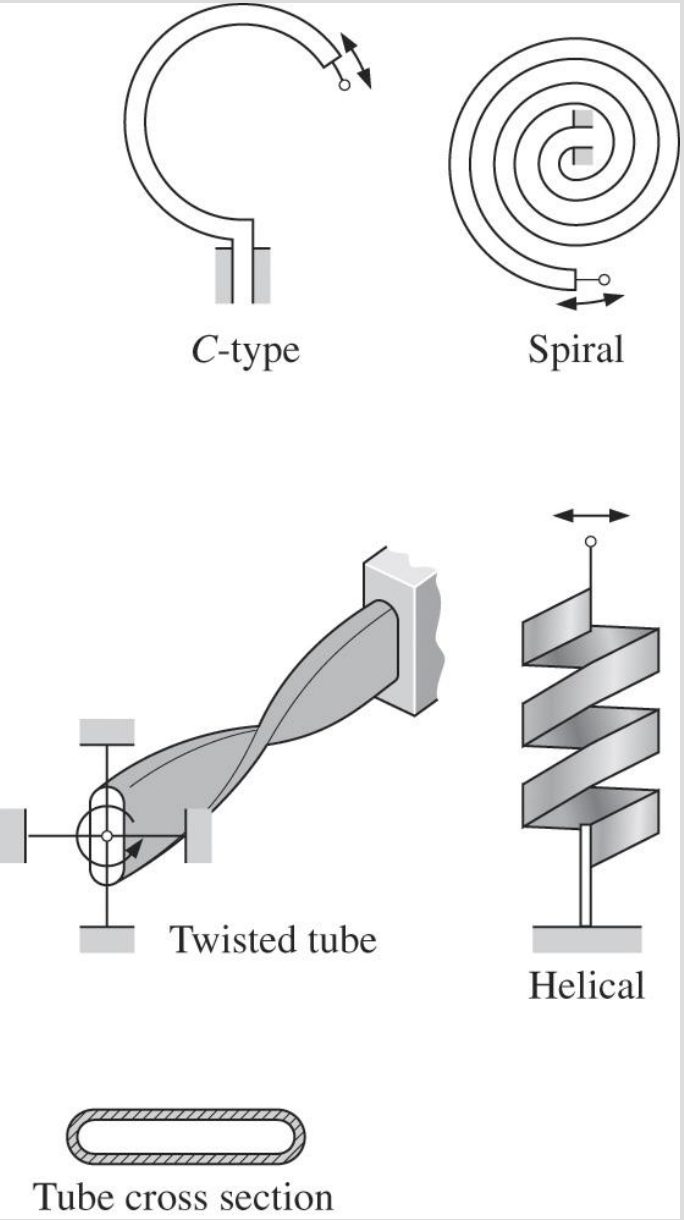
$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$



Other Pressure Measurement Devices

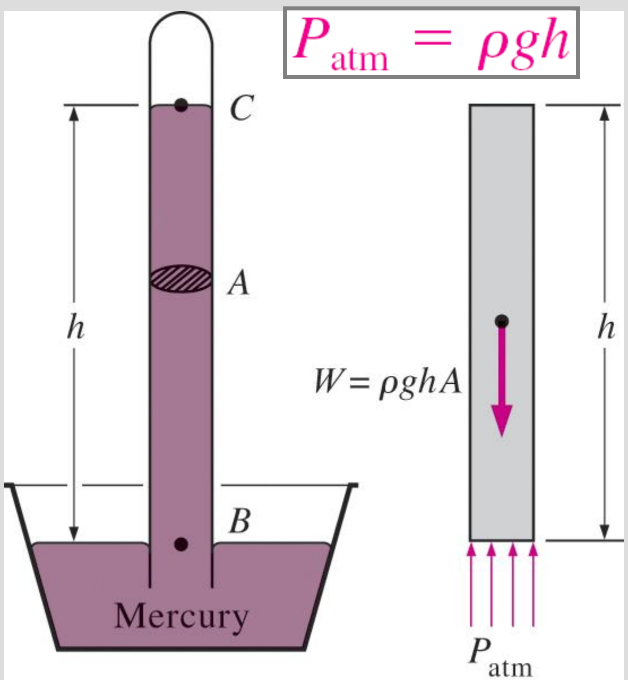
- **Bourdon tube**: Consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle.
- **Pressure transducers**: Use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance.
- Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts.
- **Strain-gage pressure transducers:** Work by having a diaphragm deflect between two chambers open to the pressure inputs.
- **Piezoelectric transducers**: Also called solid-state pressure transducers, work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure.



Various types of Bourdon tubes used to measure pressure.

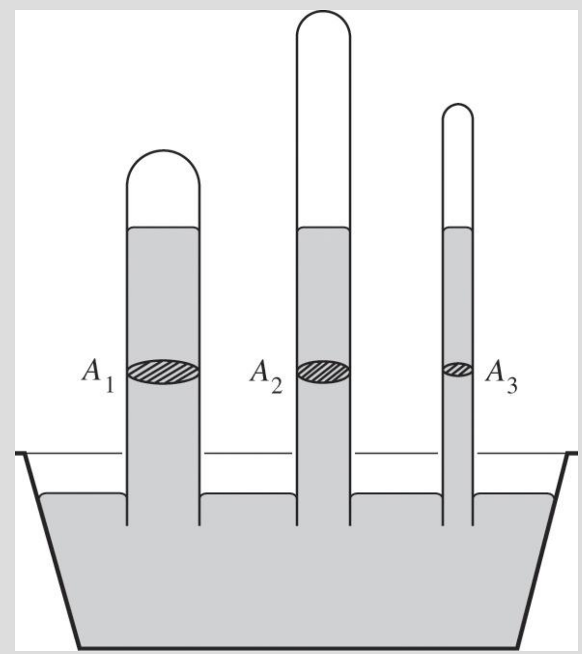
THE BAROMETER AND ATMOSPHERIC PRESSURE

- Atmospheric pressure is measured by a device called a **barometer**; thus, the atmospheric pressure is often referred to as the ***barometric pressure***.
- A frequently used pressure unit is the *standard atmosphere*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg} = 13,595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2).



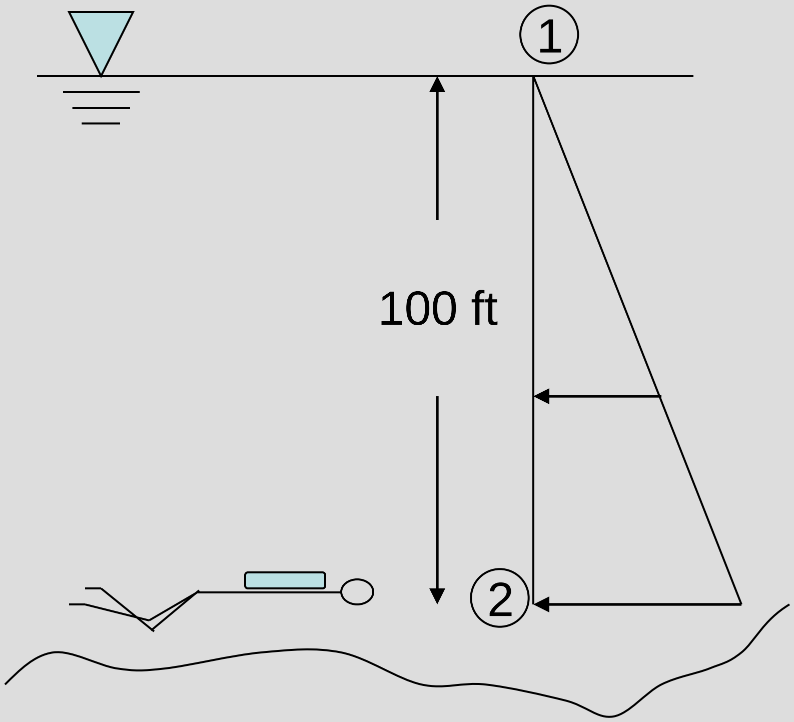
The basic barometer.

The length or the cross-sectional area of the tube has no effect on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effects.



Scuba Diving and Hydrostatic Pressure

Scuba Diving and Hydrostatic Pressure



- Pressure on diver at 100 ft?

$$P_{gage,2} = \rho g z = \left(998\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)(100\,ft)\left(\frac{1m}{3.28\,ft}\right)$$

$$= 298.5kPa\left(\frac{1atm}{101.325kPa}\right) = 2.95atm$$

$$P_{abs,2} = P_{gage,2} + P_{atm} = 2.95atm + 1atm = \boxed{3.95atm}$$

- Danger of emergency ascent?

Boyle's law

$$P_1V_1 = P_2V_2$$

$$\frac{V_1}{V_2} = \frac{P_2}{P_1} = \frac{3.95atm}{1atm} \approx 4$$

If you hold your breath on ascent, your lung volume would increase by a factor of 4, which would result in embolism and/or death.

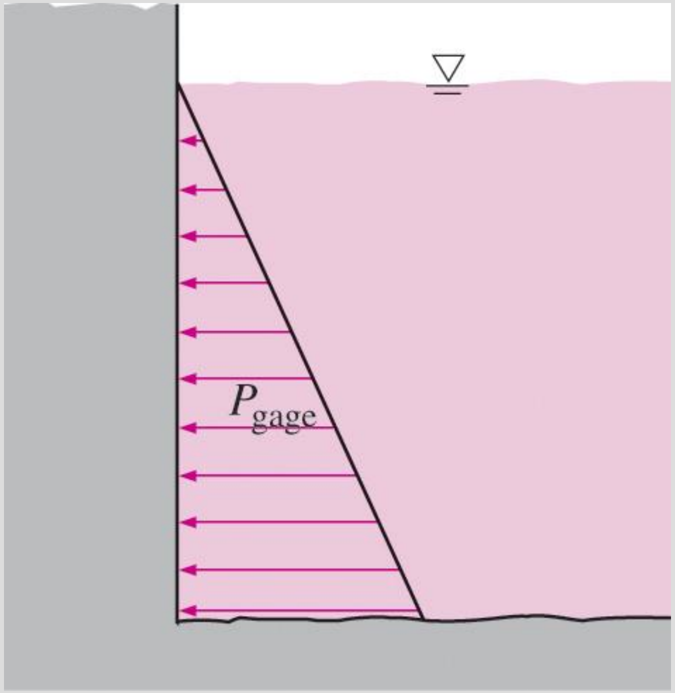
Variation of Pressure with Depth

$$\Delta P = P_2 - P_1 = \rho g \, \Delta z = \gamma_s \, \Delta z$$

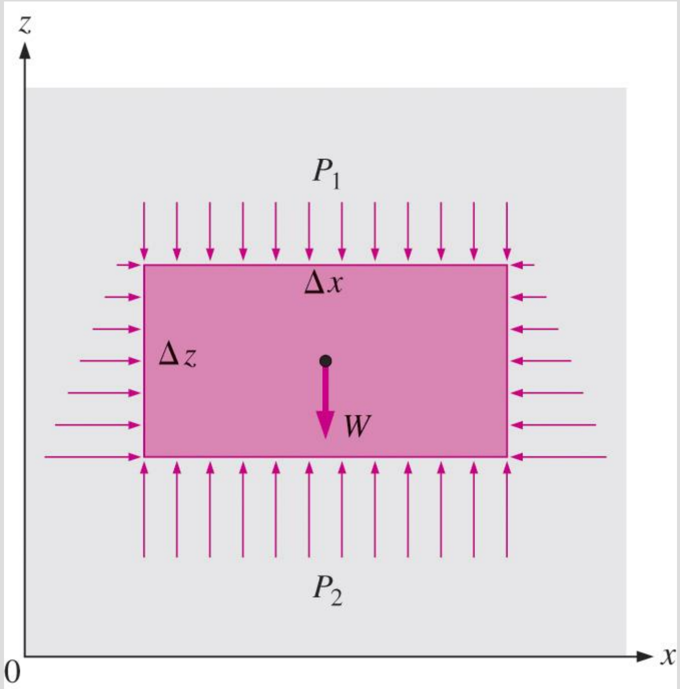
$$P = P_{\text{atm}} + \rho g h \quad \text{or} \quad P_{\text{gage}} = \rho g h$$

When the variation of density with elevation is known

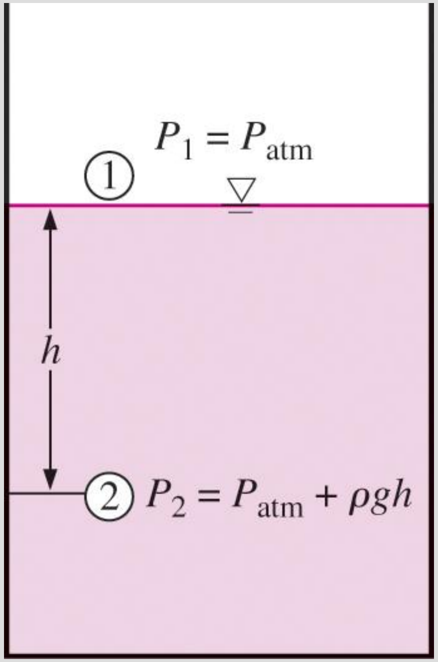
$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g \, dz$$



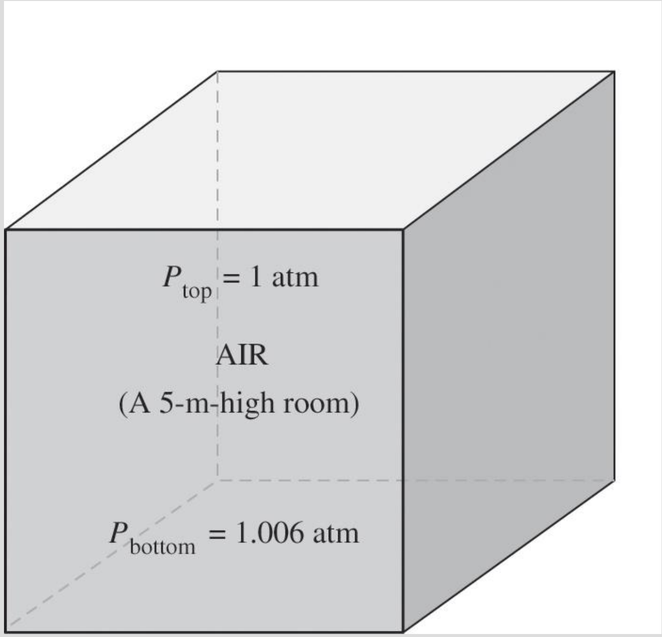
The pressure of a fluid at rest increases with depth (as a result of added weight).



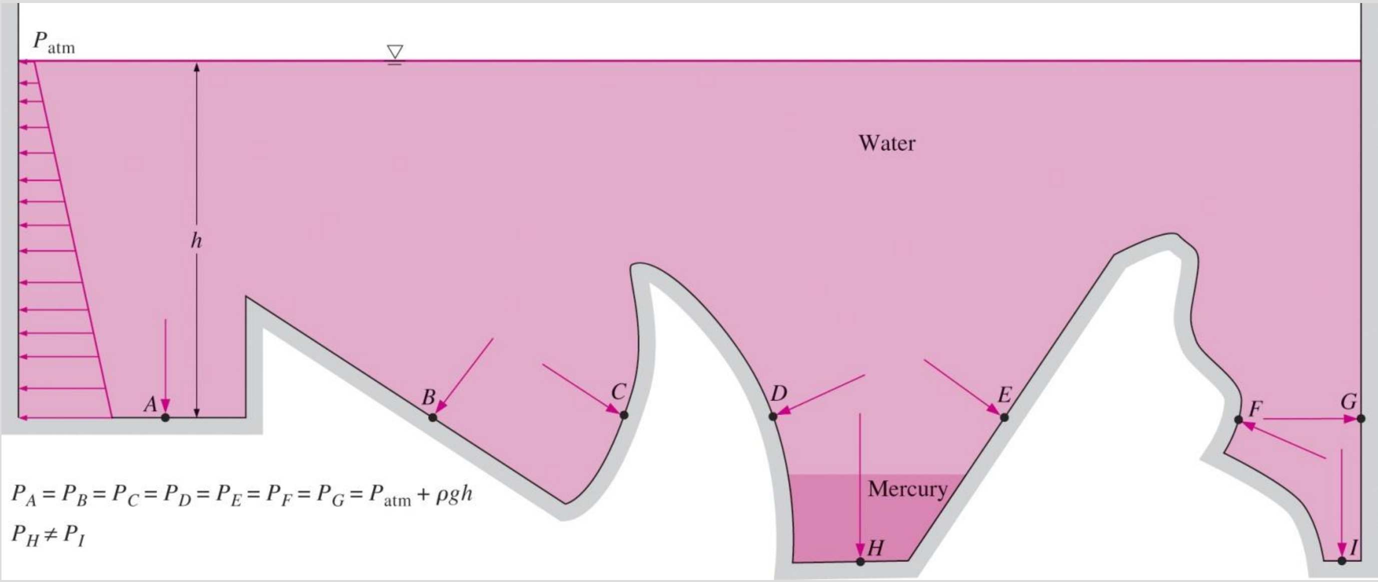
Free-body diagram of a rectangular fluid element in equilibrium.



In a room filled with a gas, the variation of pressure with height is negligible.



Pressure in a liquid at rest increases linearly with distance from the free surface.

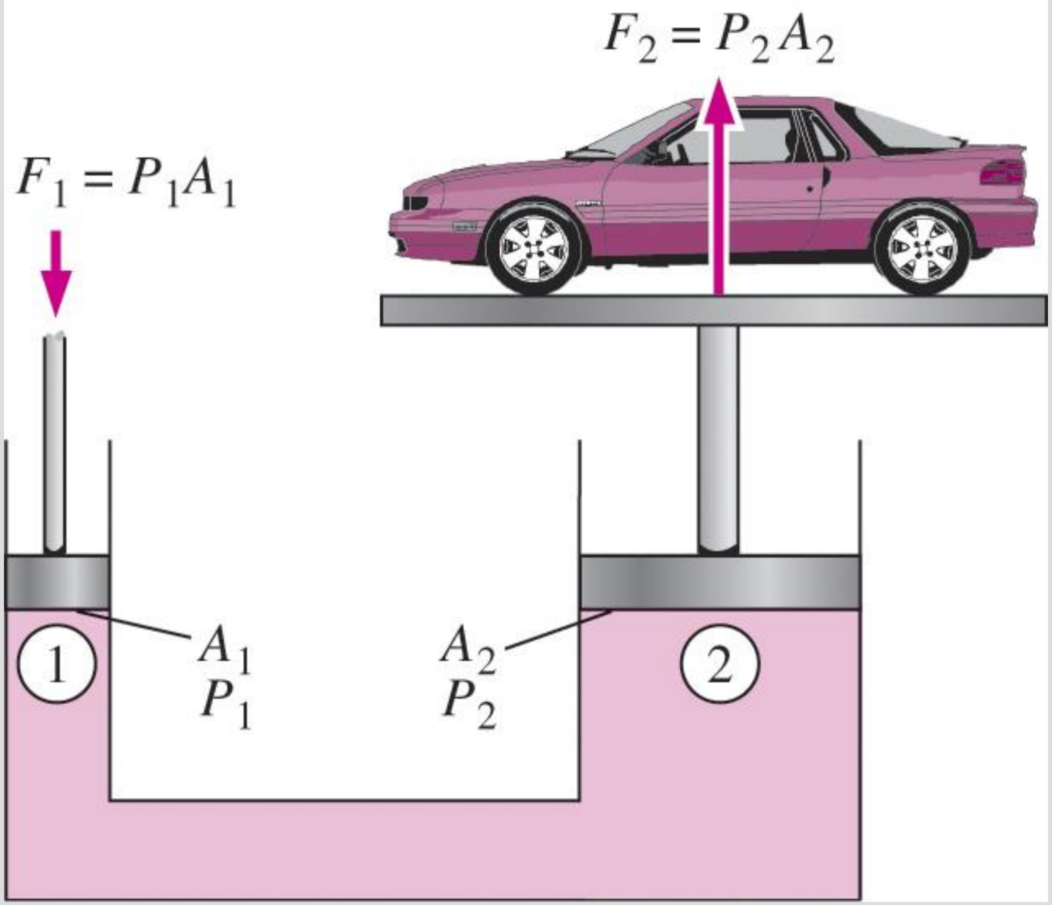


The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

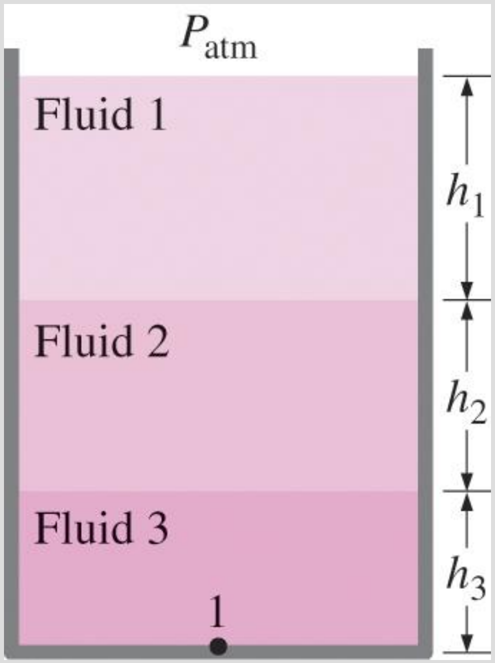
The area ratio A_2/A_1 is called the *ideal mechanical advantage* of the hydraulic lift.



Lifting of a large weight by a small force by the application of Pascal's law.

The Manometer

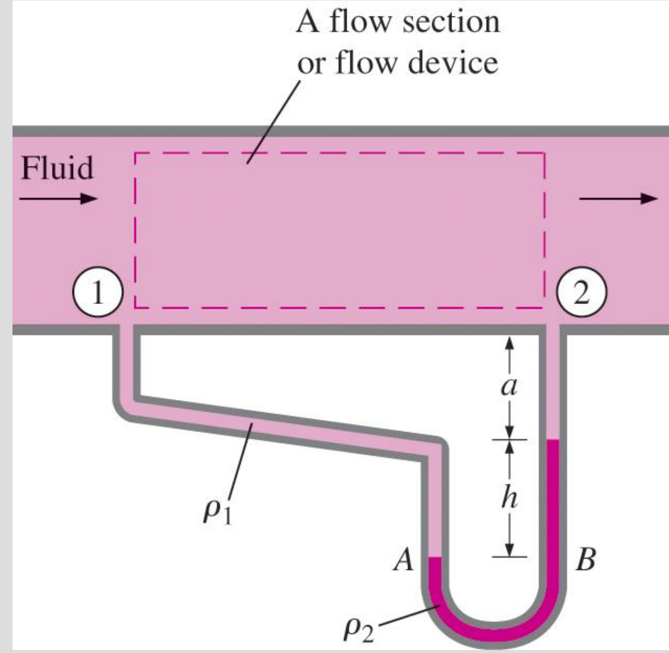
It is commonly used to measure small and moderate pressure differences. A manometer contains one or more fluids such as mercury, water, alcohol, or oil.



$$P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

In stacked-up fluid layers, the pressure change across a fluid layer of density ρ and height h is ρgh.

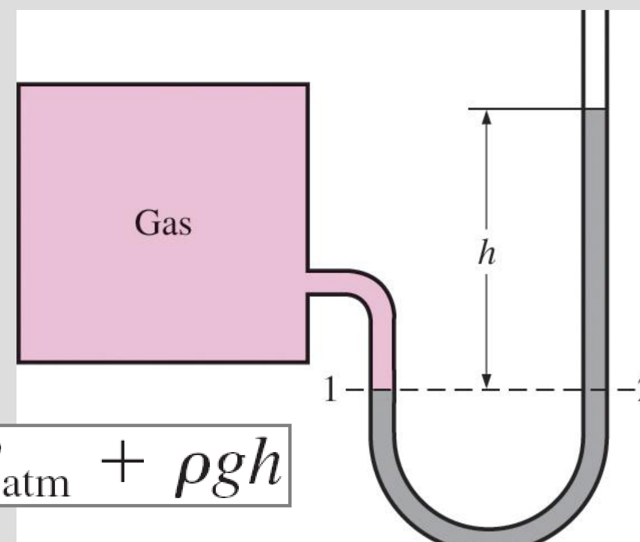
Measuring the pressure drop across a flow section or a flow device by a differential manometer.



$$P_1 + \rho_1 g(a + h) - \rho_2 gh - \rho_1 ga = P_2$$

$$P_1 - P_2 = (\rho_2 - \rho_1)gh$$

The basic manometer.

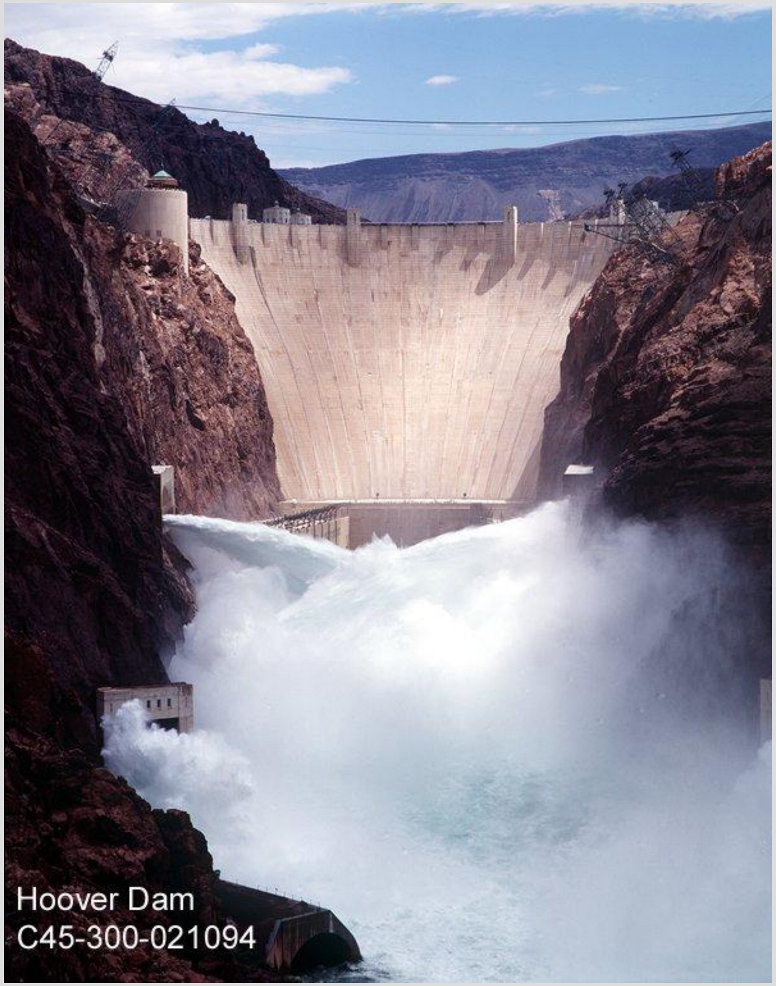


$$P_2 = P_{atm} + \rho gh$$

Force in submerged surface

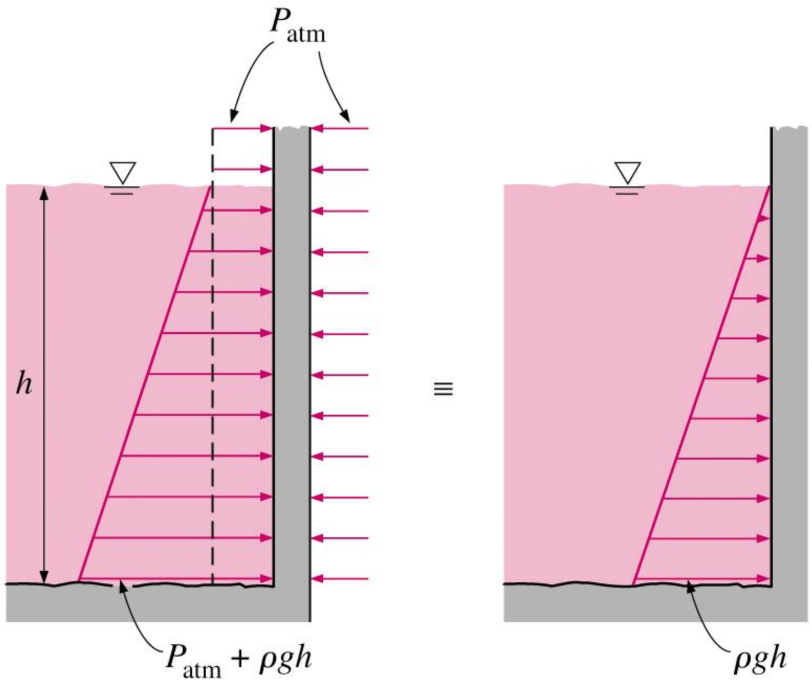
Hoover Dam

Hoover Dam



- Example of elevation head *z* converted to velocity head $V^2/2g$. We'll discuss this in more detail in Chapter 5 (Bernoulli equation).

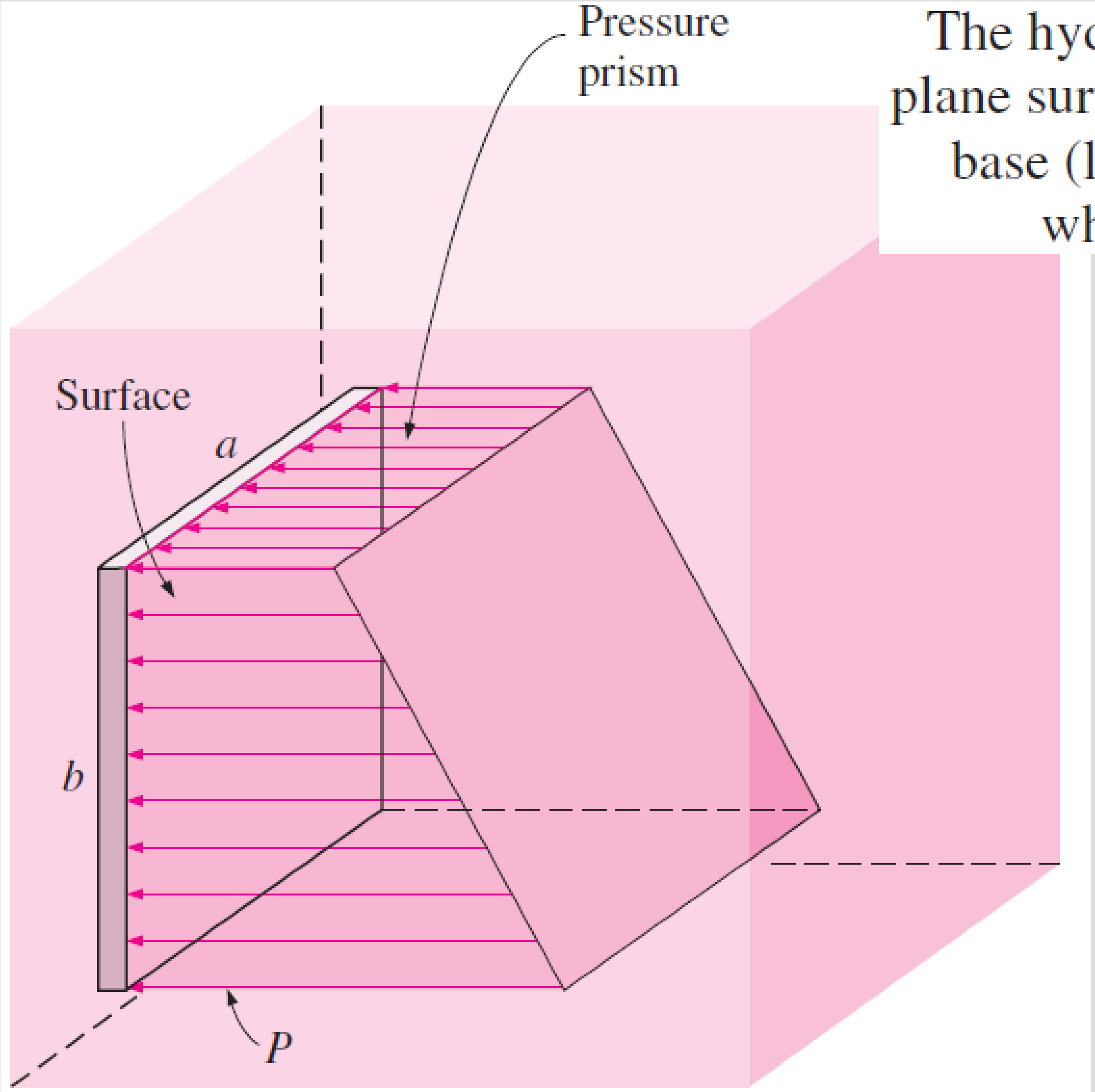
Hydrostatic Forces on Plane Surfaces



(*a*) P_{atm} considered (*b*) P_{atm} subtracted

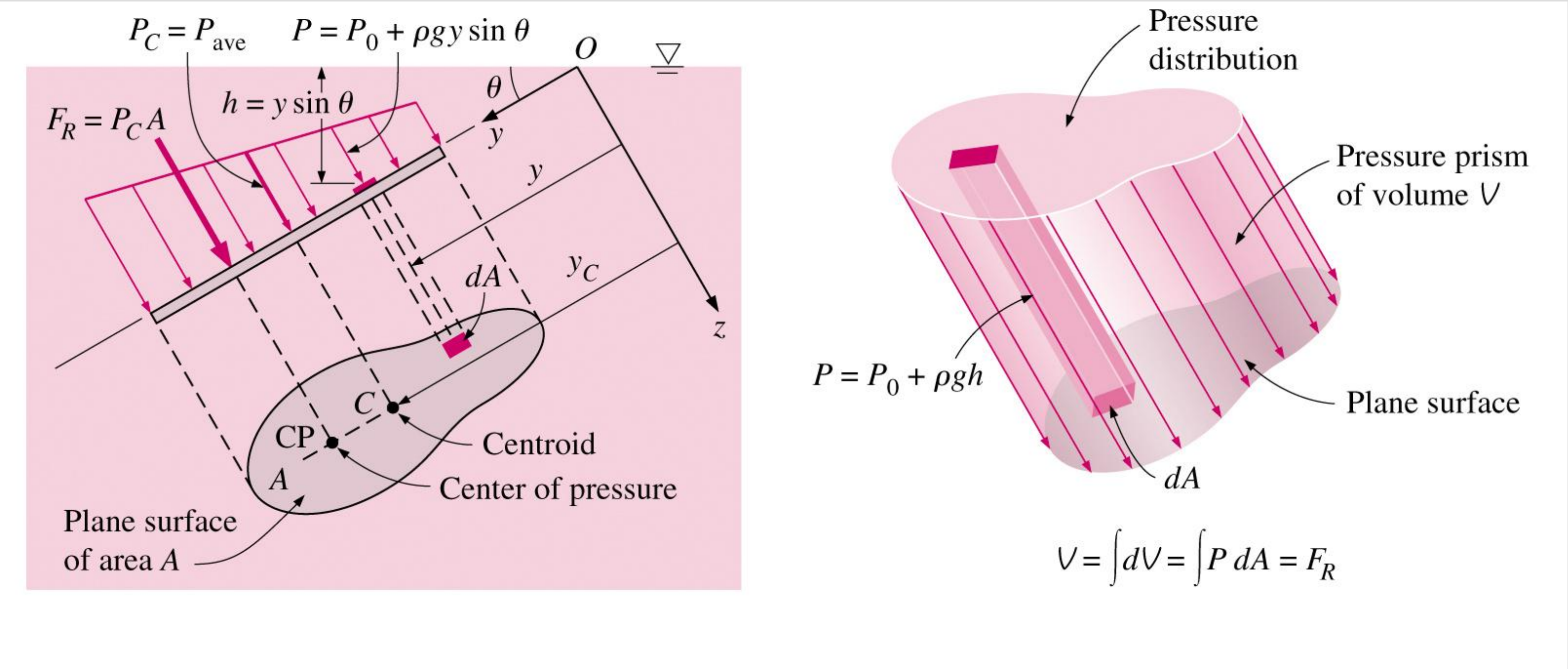
- On a *plane* surface, the hydrostatic forces form a system of parallel forces
- For many applications, magnitude and location of application, which is called **center of pressure**, must be determined.
- Atmospheric pressure P_{atm} can be neglected when it acts on both sides of the surface.

Hydrostatic Forces on Plane Surfaces



The hydrostatic forces acting on a plane surface form a volume whose base (left face) is the surface and whose height is the pressure.

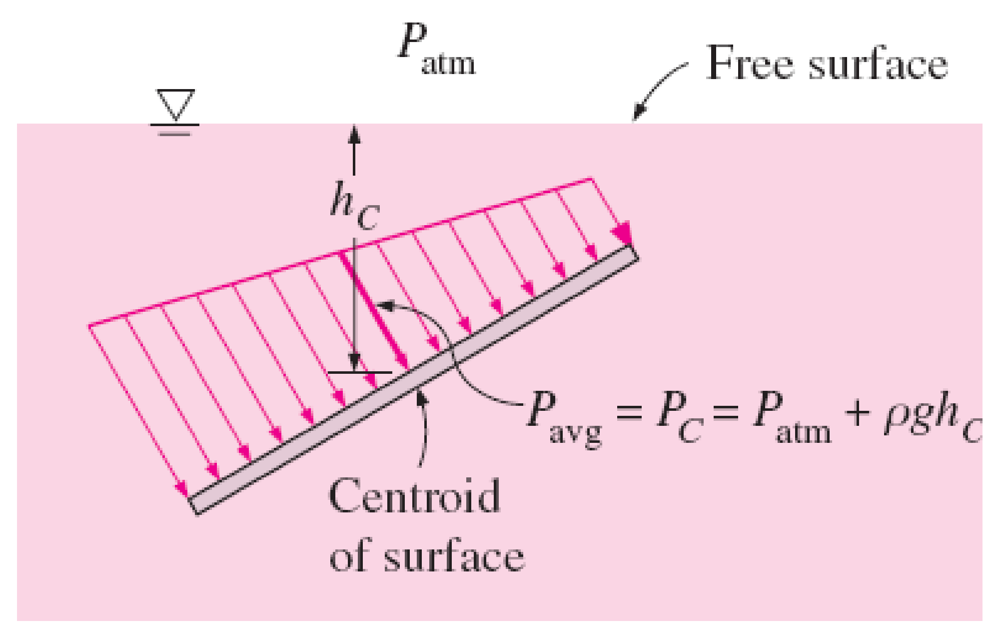
Resultant Force



The magnitude of F_R acting on a plane surface of a completely submerged plate in a homogenous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface

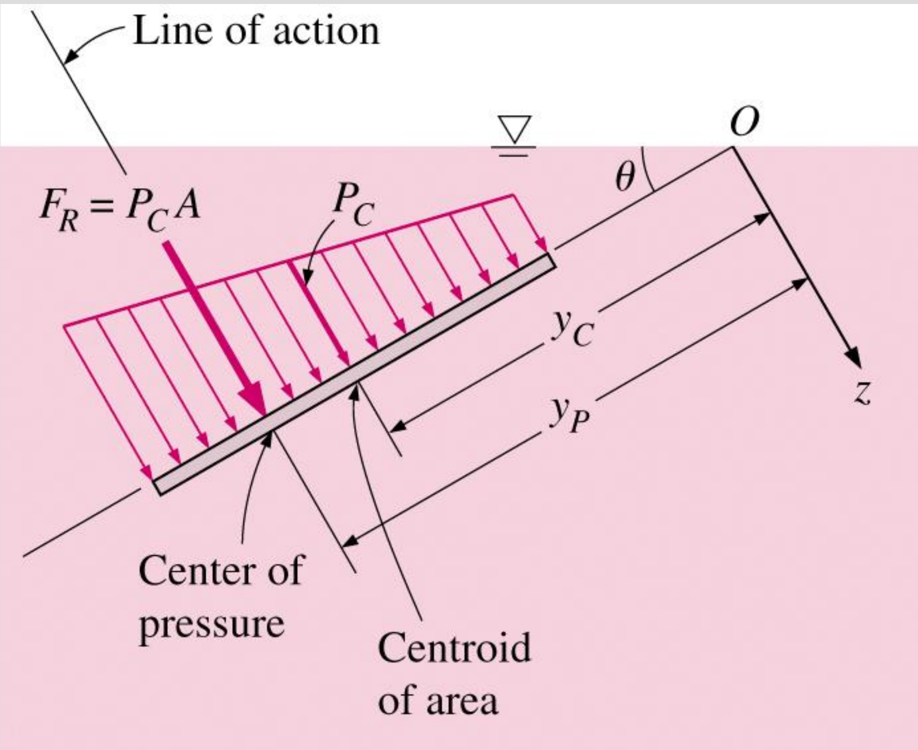
Resultant Force

$$F_R = (P_0 + \rho g y_C \sin\theta)A = (P_0 + \rho g h_C)A = P_C A = P_{avg} A$$



The pressure at the centroid of a surface is equivalent to the *average* pressure on the surface

Center of Pressure

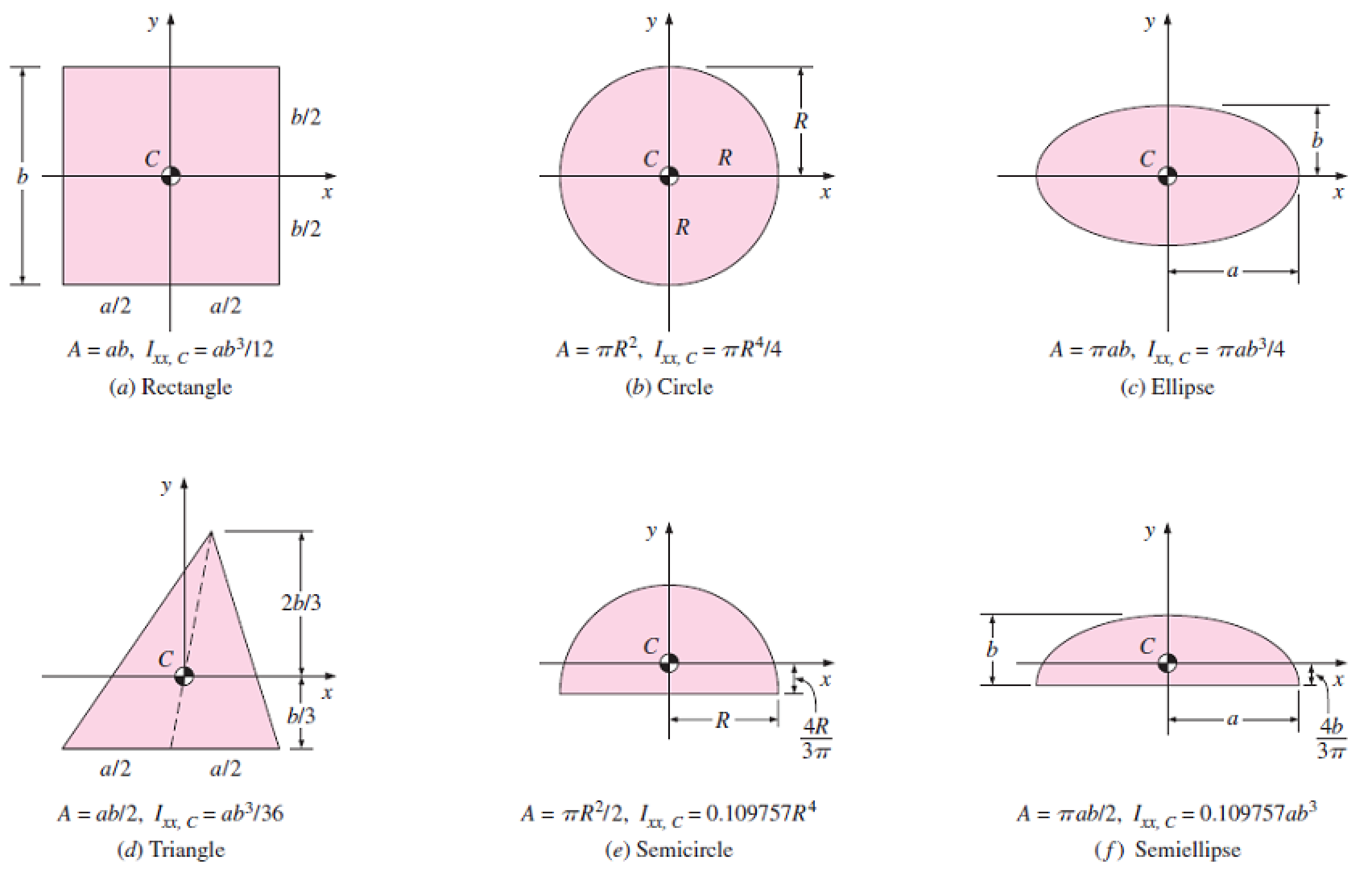


- Line of action of resultant force $F_R=P_C A$ does not pass through the centroid of the surface. In general, it lies underneath where the pressure is higher.
- Vertical location of **Center of Pressure** is determined by equation the moment of the resultant force to the moment of the distributed pressure force.

$$y_p = y_C + \frac{I_{xx,C}}{y_c A}$$

- $I_{xx,C}$ is tabulated for simple geometries.

Centroide y momento centroidal

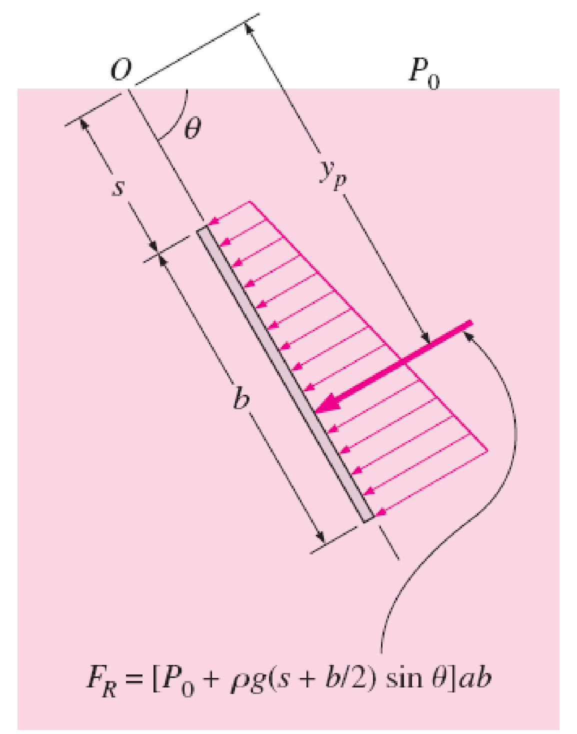


Special Case: Submerged Rectangular Plate

Hydrostatic force acting on the top surface of a submerged tilted rectangular plate.

$$y_P = s + \frac{b}{2} + \frac{ab^3/12}{[s + b/2 + P_0/(\rho g \sin\theta)]ab}$$

$$= s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin\theta)]}$$

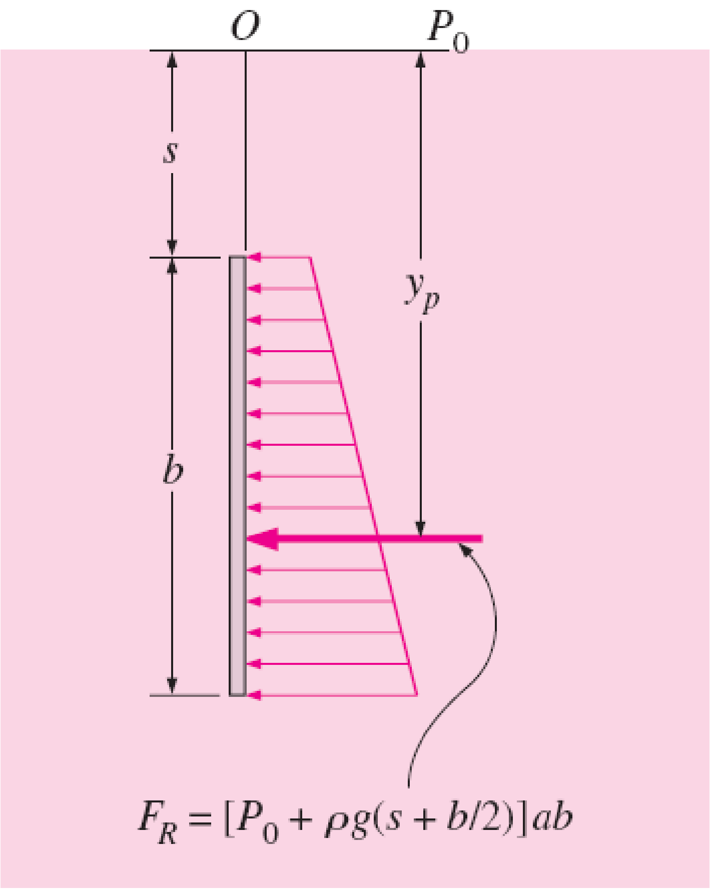


(*a*) Tilted plate

Tilted rectangular plate: $F_R = P_C A = [P_0 + \rho g(s + b/2)\sin\theta]ab$

Tilted rectangular plate ($s = 0$): $F_R = [P_0 + \rho g(b\sin\theta)/2]ab$

Special Case: Submerged Rectangular Plate



(b) Vertical plate

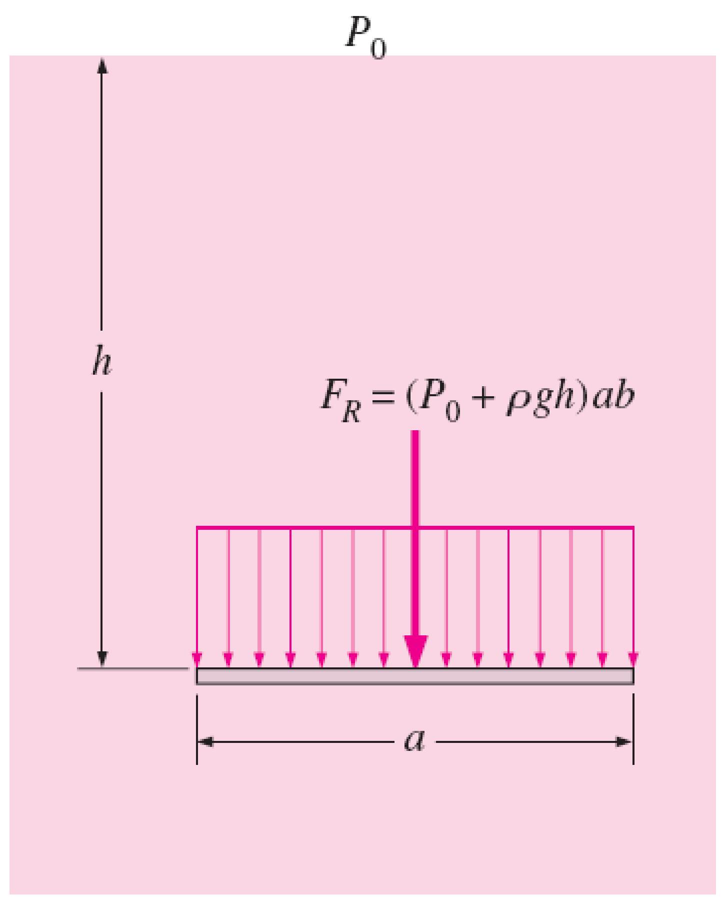
Hydrostatic force acting on the top surface of a submerged vertical rectangular plate.

Vertical rectangular plate: $F_R = [P_0 + \rho g(s + b/2)]ab$

Vertical rectangular plate ($s = 0$): $F_R = (P_0 + \rho g b/2)ab$

Special Case: Submerged Rectangular Plate

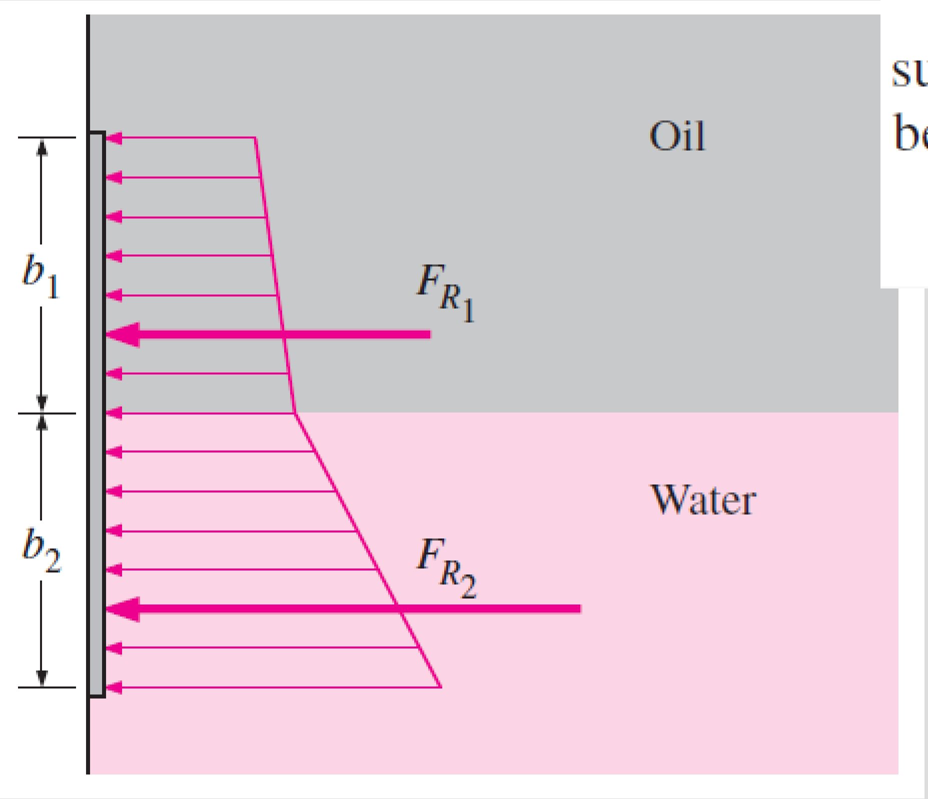
Hydrostatic force acting on the top surface of a submerged horizontal rectangular plate.



(c) Horizontal plate

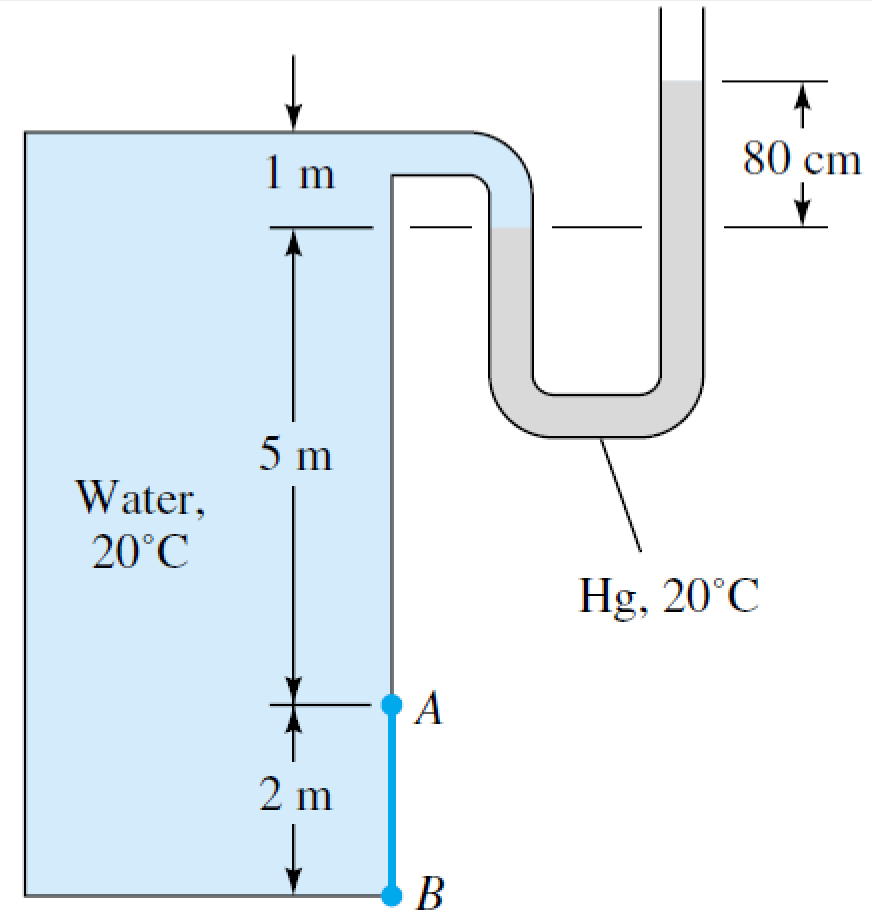
Horizontal rectangular plate: $F_R = (P_0 + \rho g h)ab$

Special Case: Surface submerged in multilayered fluids



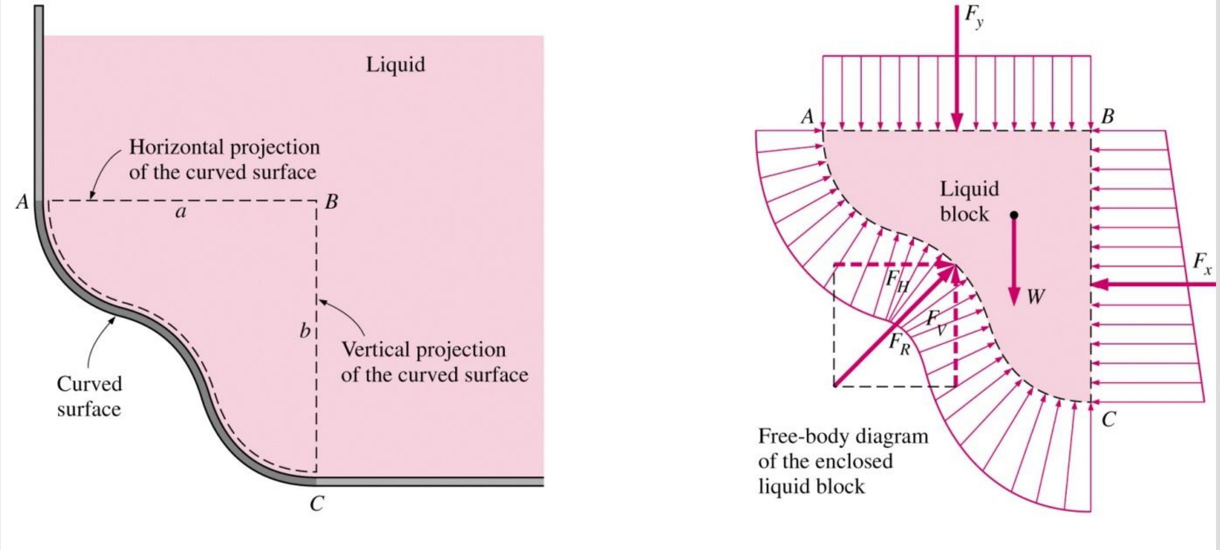
The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.

Quiz



The Water tank in the Figure is pressurized, as shown by the mercury-manometer reading. Determine the hidrostatic force on the gate AB with 3m wide.

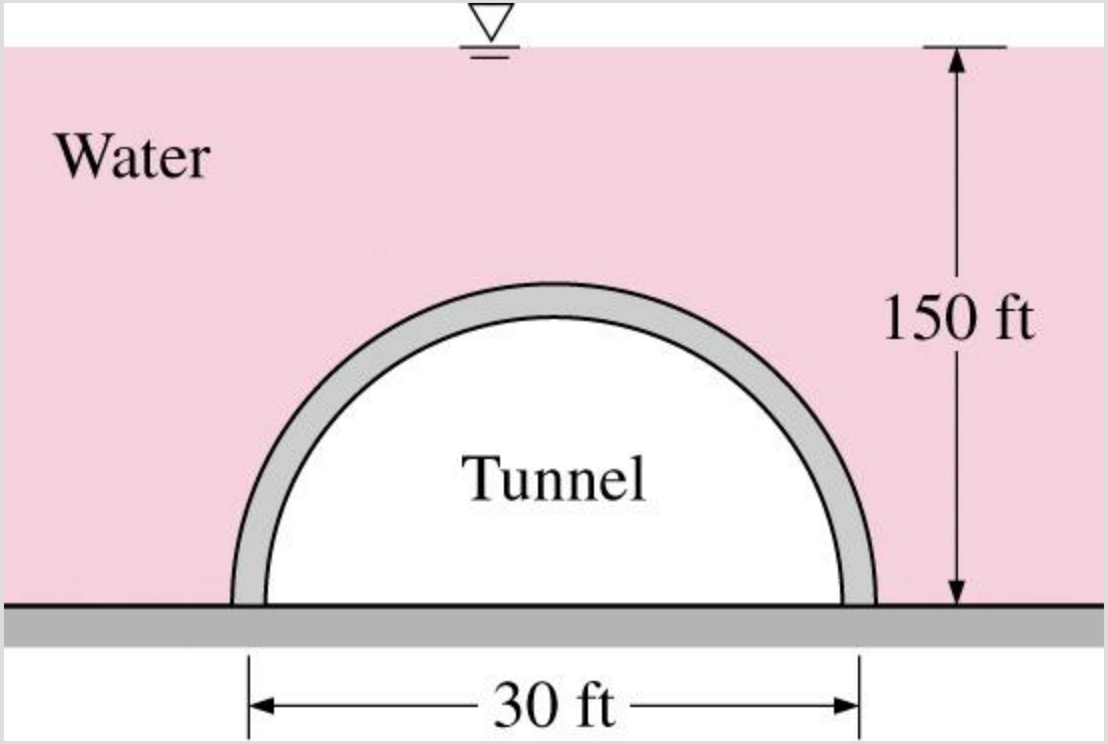
Hydrostatic Forces on Curved Surfaces



- F_R on a curved surface is more involved since it requires integration of the pressure forces that change direction along the surface.
- Easiest approach: determine horizontal and vertical components F_H and F_V separately.

Hydrostatic Forces on Curved Surfaces, example

A semicircular 30 ft diameter tunnel is to be built under a 150 ft deep, 800 ft long lake, as shown in Figure. Determine the total hydrostatic force acting on the roof of the tunnel.



Hydrostatic Forces on Curved Surfaces, example

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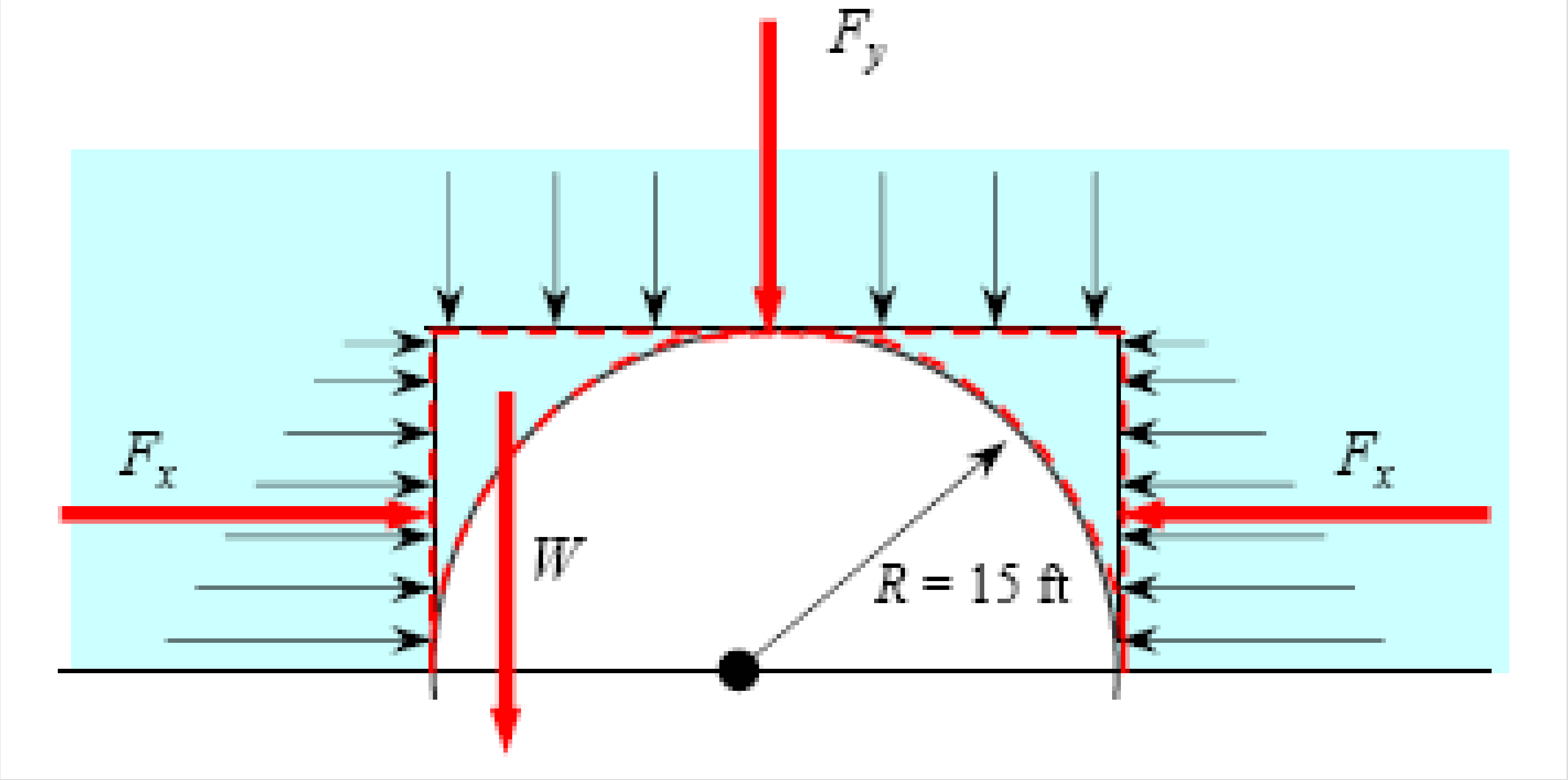
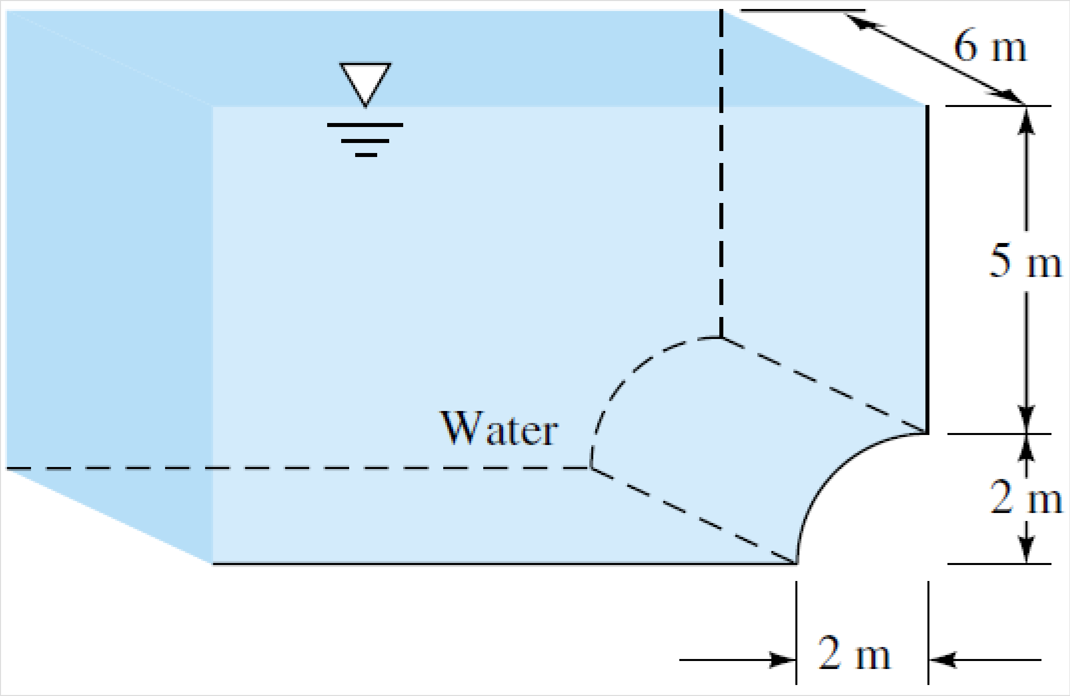


Hydrostatic Forces on Curved Surfaces

- Horizontal force component on curved surface: $F_H = F_x$. Line of action on vertical plane gives y coordinate of center of pressure on curved surface.
- Vertical force component on curved surface: $F_V = F_y + W$, where W is the weight of the liquid in the enclosed block $W = \rho g V$. x coordinate of the center of pressure is a combination of line of action on horizontal plane (centroid of area) and line of action through volume (centroid of volume).
- Magnitude of force $F_R = (F_H^2 + F_V^2)^{1/2}$
- Angle of force is $\alpha = tan^{-1}(F_V/F_H)$

Exercice:

Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in the figure.

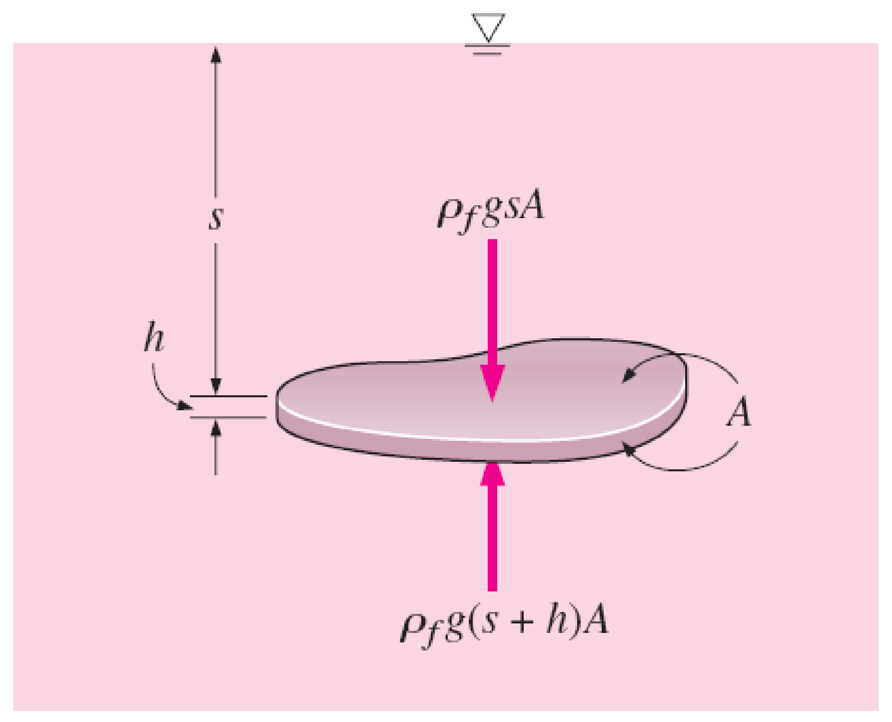


HIDROSTATIC

Buoyancy and rigid-body motion

Buoyancy and Stability

Buoyant force: The upward force a fluid exerts on a body immersed in it. The buoyant force is caused by the increase of pressure with depth in a fluid.



A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface.

The buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.

For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface.

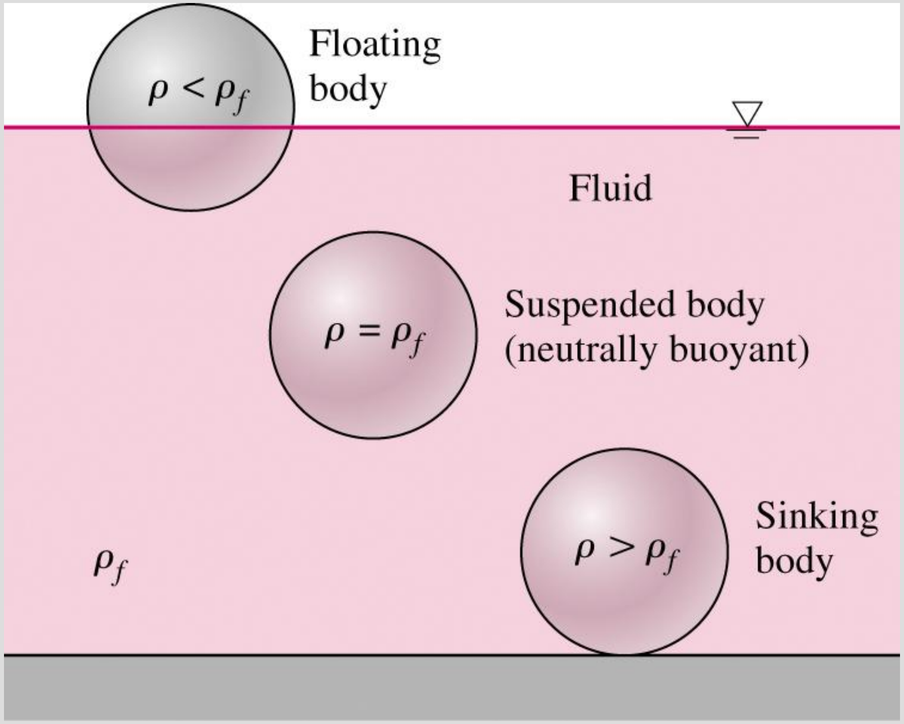
It is also independent of the density of the solid body.

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f g s A = \rho_f g h A = \rho_f g V$$

Buoyancy and Stability

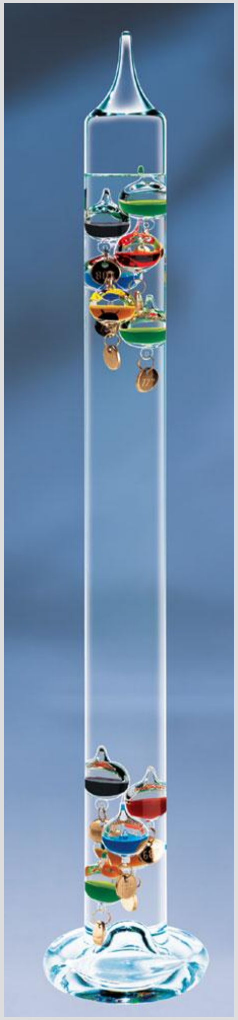
- Buoyancy is due to the fluid displaced by a body. $F_B = \rho_f g V$.
- **Archimedes principal** : The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

Buoyancy and Stability



- Buoyancy force F_B is equal only to the displaced volume $\rho_f g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{body} < \rho_{fluid}$: Floating body
  2. $\rho_{body} = \rho_{fluid}$: Neutrally buoyant
  3. $\rho_{body} > \rho_{fluid}$: Sinking body

Example: Galilean Thermometer



- Galileo's thermometer is made of a sealed glass cylinder containing a clear liquid.
- Suspended in the liquid are a number of weights, which are sealed glass containers with colored liquid for an attractive effect.
- As the liquid changes temperature it changes density and the suspended weights rise and fall to stay at the position where their density is equal to that of the surrounding liquid.
- If the weights differ by a very small amount and ordered such that the least dense is at the top and most dense at the bottom they can form a temperature scale.

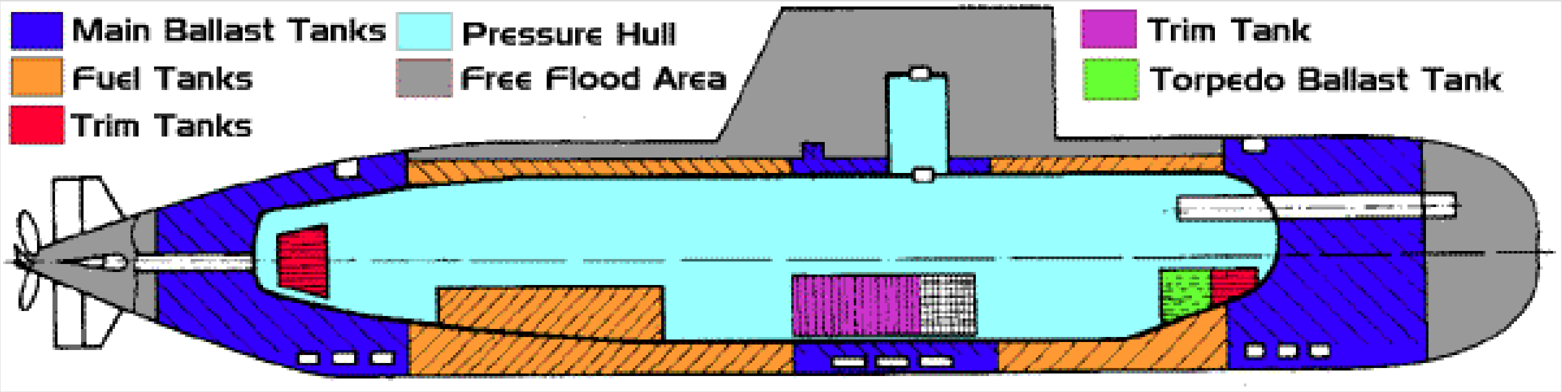
Example: Floating Drydock

Auxiliary Floating Dry Dock Resolute (AFDM-10) partially submerged

Submarine undergoing repair work on board the AFDM-10

Using buoyancy, a submarine with a displacement of 6,000 tons can be lifted!

Example: Submarine Buoyancy and Ballast



- Submarines use both static and dynamic depth control. Static control uses ballast tanks between the pressure hull and the outer hull. Dynamic control uses the bow and stern planes to generate trim forces.

Example: Submarine Buoyancy and Ballast

Normal surface trim

SSN 711 nose down after accident which damaged fore ballast tanks

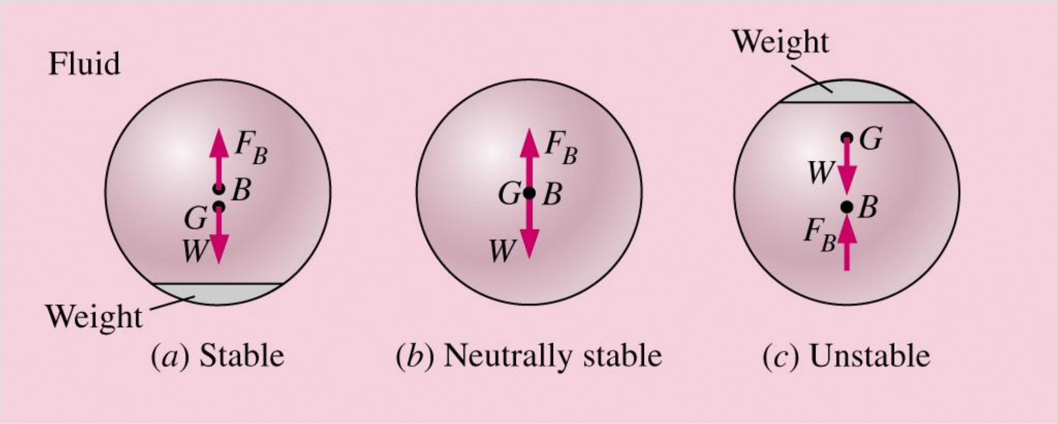
Example: Submarine Buoyancy and Ballast

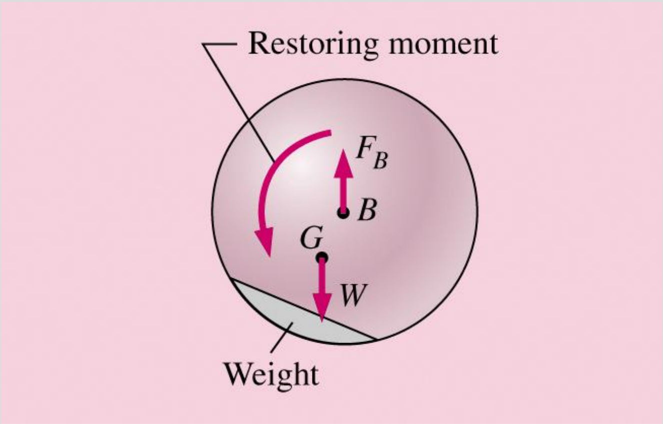
Damage to SSN 711 (USS San Francisco) after running aground on 8 January 2005.

Example: Submarine Buoyancy and Ballast

Ballast Control Panel: Important station for controlling depth of submarine

Stability of Immersed Bodies

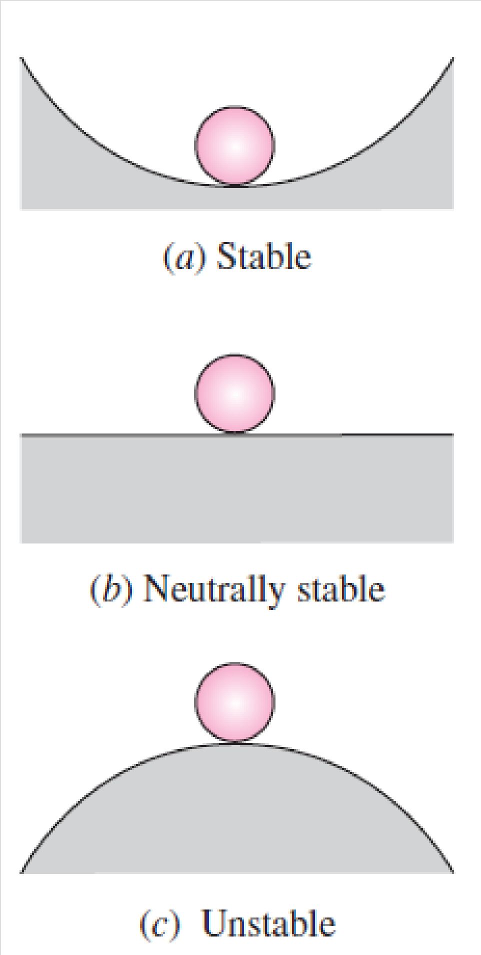




- Rotational stability of immersed bodies depends upon relative location of *center of gravity G* and *center of buoyancy B.*
  - ✓ *G* below *B*: stable
  - ✓ *G* above *B*: unstable
  - ✓ *G* coincides with *B*: neutrally stable.

Stability of Floating Bodies

An important application of the buoyancy concept is the assessment of the stability of immersed and floating bodies with no external attachments. This topic is of great importance in the design of ships and submarines.

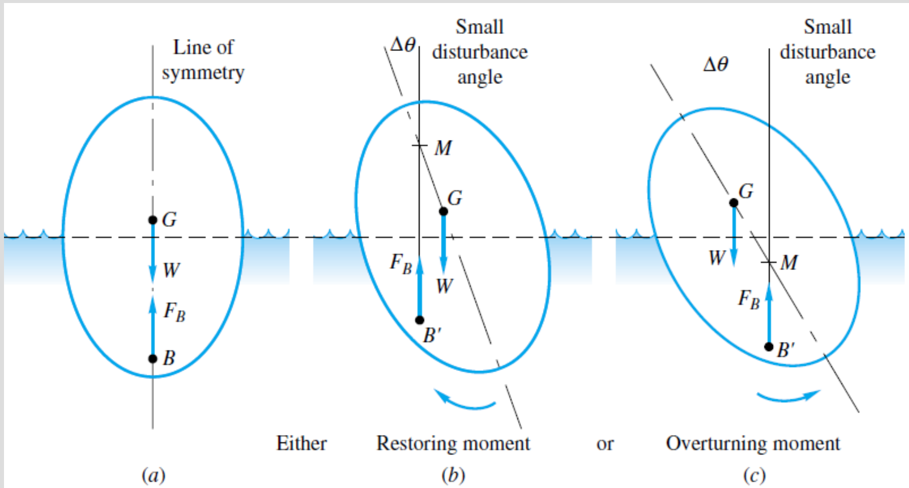


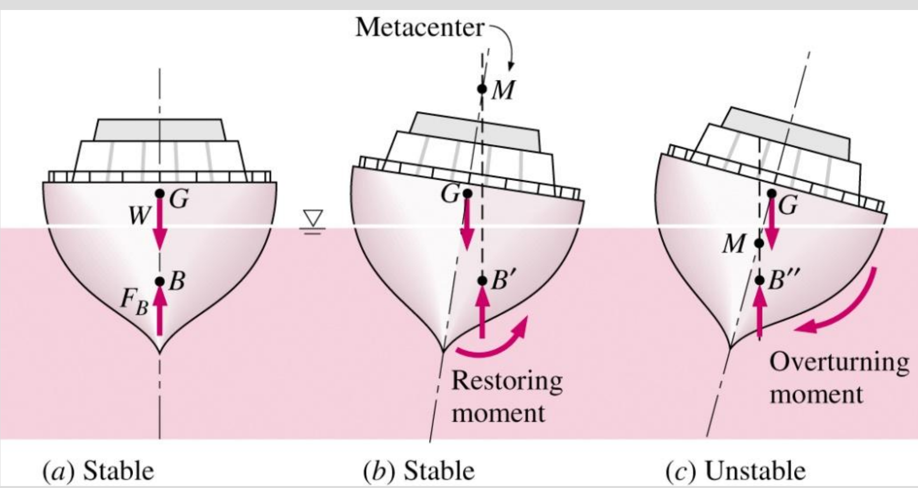
Stable: Since any small disturbance (someone moves the ball to right or left) generates a restoring force (due to gravity) that returns it to its initial position

Neutral stable: Because if someone moves the ball to the right or left, it would stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away

Unstable: Is a situation in which the ball may be at rest at the moment, but any disturbance, even an infenitesimal one, causes the ball to roll off the hill-it does not return to its original position; rather it *diverges* from it.

Stability of Floating Bodies





- If body is bottom heavy (*G* lower than *B*), it is always stable.
- Floating bodies can be stable when *G* is higher than *B* due to shift in location of center buoyancy and creation of restoring moment.
- Measure of stability is the metacentric height G*M*. If *GM*>1, ship is stable.

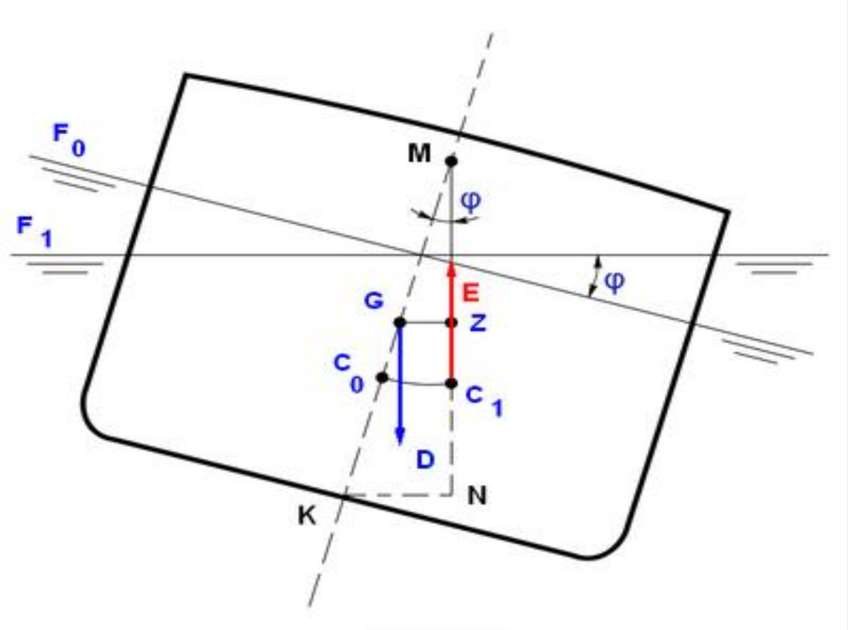
Stability of Floating Bodies

The **metacentric height** is a measurement of the static stability of a floating body. It is calculated as the distance between the centre of gravity of a ship and its metacentre (GM). A larger metacentric height implies greater stability against overturning. Metacentric height also has implication on the natural period of rolling of a hull, with very large metacentric heights being associated with shorter periods of roll which are uncomfortable for passengers. Hence, a sufficiently high, but not excessively high metacentric height is considered ideal for passenger ships.

Stable: $\overline{KM} > \overline{KG}$

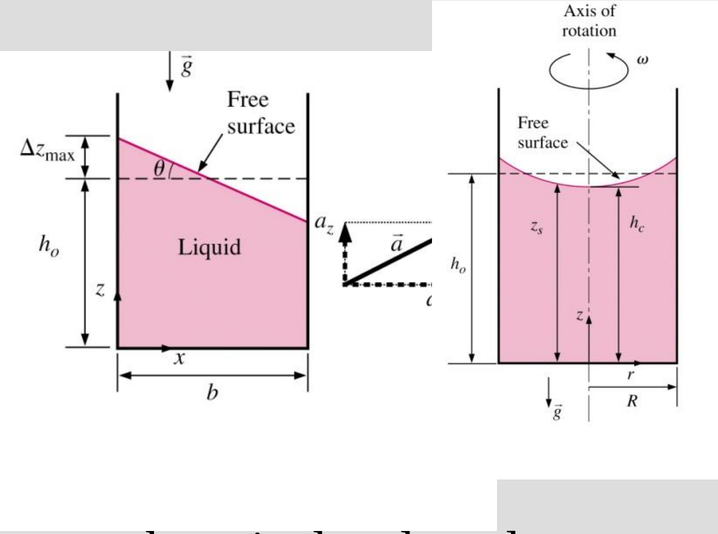
Unstable: $\overline{KM} < \overline{KG}$

Indifferent: $\overline{KM} = \overline{KG}$



Rigid-Body Motion

- There are special cases where a body of fluid can undergo rigid-body motion: linear acceleration, and rotation of a cylindrical container.

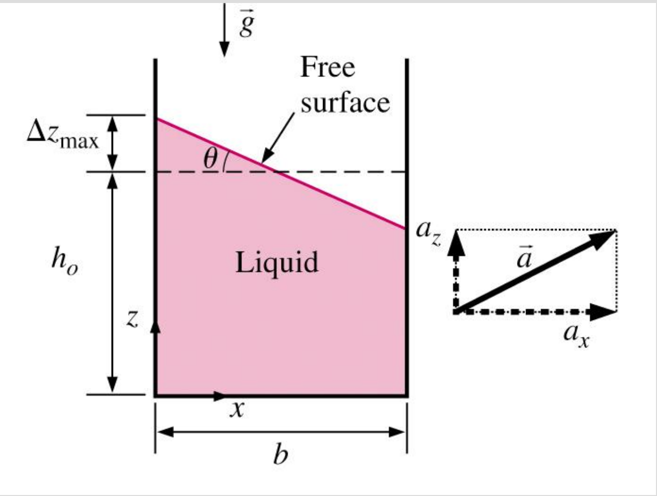


- In these cases, no shear is developed.
- Newton's 2nd law of motion can be used to derive an **equation of motion** for a fluid that acts as a rigid body

$$\nabla P + \rho g \vec{k} = -\rho \vec{a}$$

- In Cartesian coordinates: $\dfrac{\partial P}{\partial x} = -\rho a_x, \dfrac{\partial P}{\partial y} = -\rho a_y, \dfrac{\partial P}{\partial z} = -\rho \left(g + a_x \right)$

Linear Acceleration



Container is moving on a straight path

$$a_x \neq 0, a_y = a_z = 0$$

$$\frac{\partial P}{\partial x} = \rho a_x, \frac{\partial P}{\partial y} = 0, \frac{\partial P}{\partial z} = -\rho g$$

Total differential of P

$$dP = -\rho a_x dx - \rho g dz$$

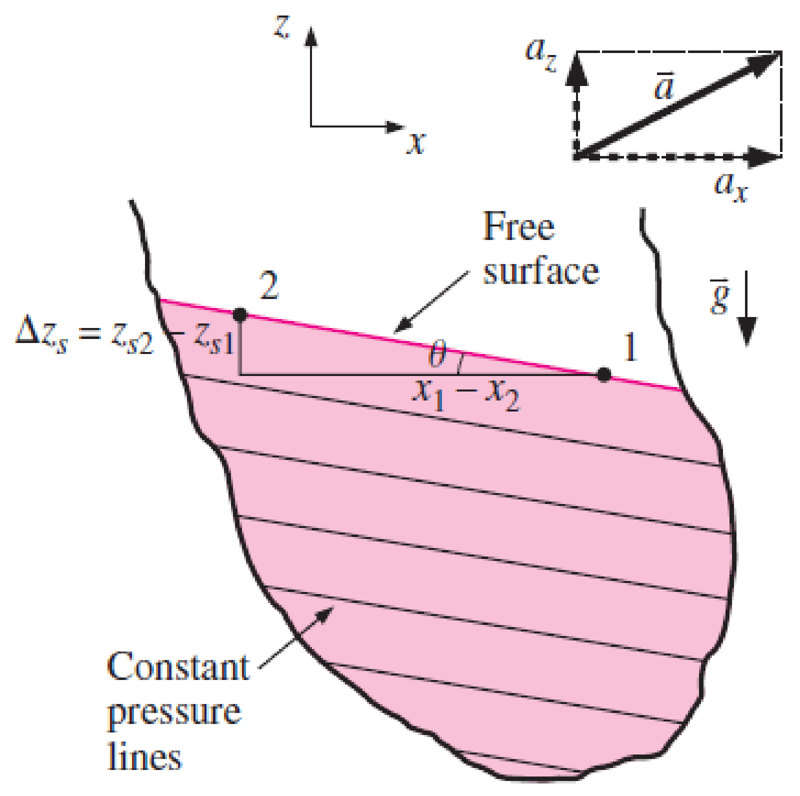
Pressure difference between 2 points

$$P_2 - P_1 = -\rho a_x \left(x_2 - x_1\right) - \rho g \left(z_2 - z_1\right)$$

Find the rise by selecting 2 points on free surface $P_2 = P_1$

$$\Delta z_s = z_{s2} - z_{s1} = -\frac{a_x}{g}\left(x_2 - x_1\right)$$

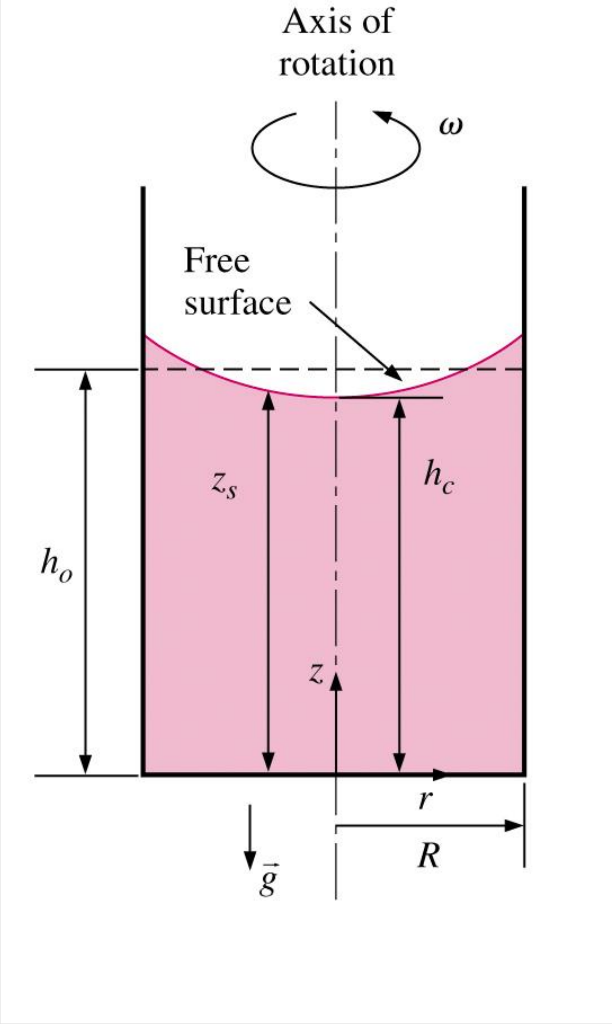
Linear Acceleration



Surfaces of constant pressure: $$\frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = \text{constant}$$

Slope of isobars: $$\text{Slope} = \frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$

Rotation in a Cylindrical Container



Container is rotating about the z-axis

$$a_r = -r\omega^2, a_\theta = a_z = 0$$

$$\frac{\partial P}{\partial r} = \rho r\omega^2, \frac{\partial P}{\partial \theta} = 0, \frac{\partial P}{\partial z} = -\rho g$$

Total differential of P

$$dP = \rho r\omega^2 dr - \rho g dz$$

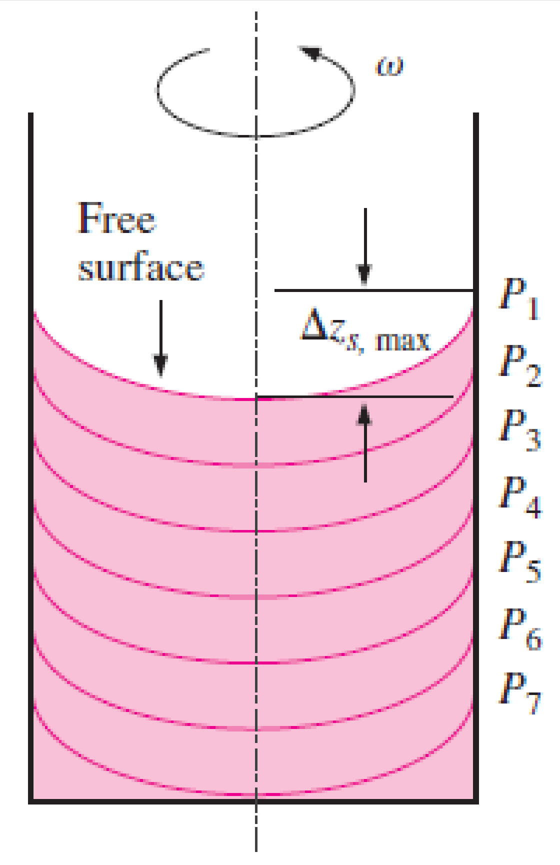
On an isobar, dP = 0

$$\frac{dz_{isobar}}{dr} = \frac{r\omega^2}{g} \rightarrow z_{isobar} = \frac{\omega^2}{2g} r^2 + C_1$$

Equation of the free surface

$$z_s = h_0 - \frac{\omega^2}{4g}\left(R^2 - 2r^2\right)$$

Rotation in a Cylindrical Container



Surfaces of constant pressure: $$z_{isobar} = \frac{\omega^2}{2g} r^2 + C_1$$

Rotation in a Cylindrical Container

The volume of a cylindrical shell element of radius r, height z_s, and thickness dr is $dV = 2\pi r z_s\, dr$. Then the volume of the paraboloid formed by the free surface is

$$V = \int_{r=0}^{R} 2\pi z_s r\, dr = 2\pi \int_{r=0}^{R} \left(\frac{\omega^2}{2g} r^2 + h_c\right) r\, dr = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c\right)$$

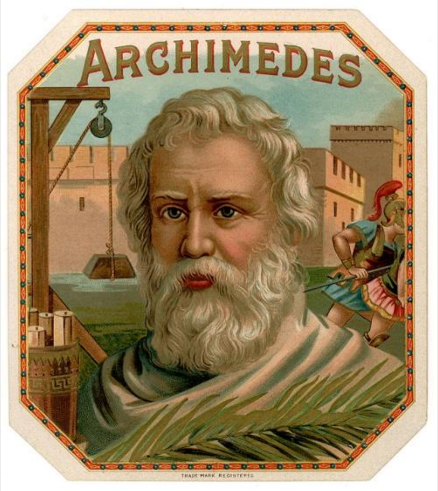
Since mass is conserved and density is constant, this volume must be equal to the original volume of the fluid in the container, which is

$$V = \pi R^2 h_0$$

where h_0 is the original height of the fluid in the container with no rotation.

Examples of Archimedes Principle

The Golden Crown of Hiero II, King of Syracuse



- Archimedes, 287-212 B.C.
- Hiero, 306-215 B.C.
- Hiero learned of a rumor where the goldsmith replaced some of the gold in his crown with silver. Hiero asked Archimedes to determine whether the crown was pure gold.
- Archimedes had to develop a nondestructive testing method

The Golden Crown of Hiero II, King of Syracuse

- The weight of the crown and nugget are the same in air: $W_c = \rho_c V_c = W_n = \rho_n V_n$.
- If the crown is pure gold, $\rho_c = \rho_n$ which means that the volumes must be the same, $V_c = V_n$.
- In water, the buoyancy force is $B = \rho_{H2O} V$.
- If the scale becomes unbalanced, this implies that the $V_c \neq V_n$, which in turn means that the $\rho_c \neq \rho_n$
- Goldsmith was shown to be a fraud!

Hydrostatic Bodyfat Testing

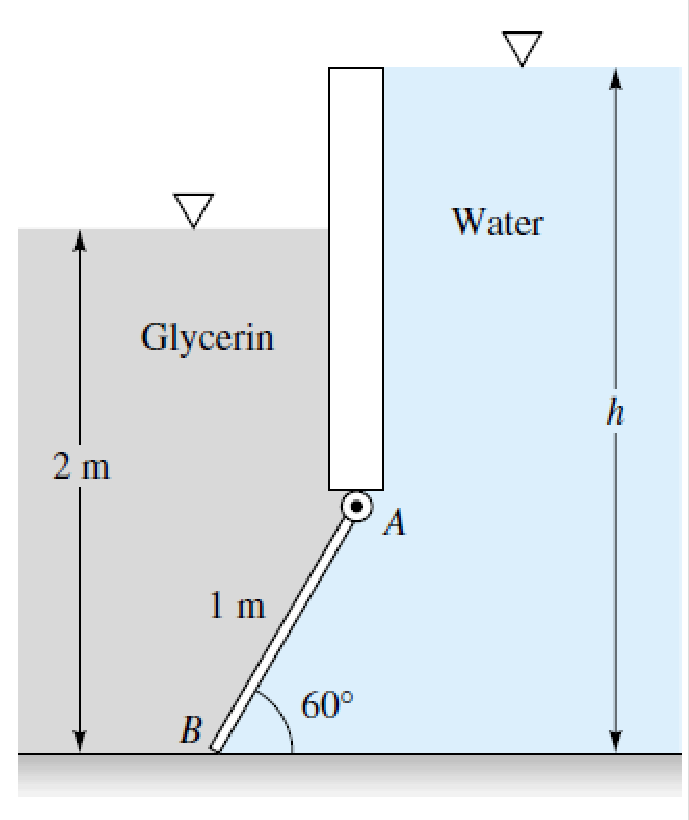
- What is the best way to measure body fat?
- Hydrostatic Bodyfat Testing using Archimedes Principle!
- Process
  - ✓ Measure body weight $W=\rho_{body}V$
  - ✓ Get in tank, expel all air, and measure apparent weight W_a
  - ✓ Buoyancy force $B = W\text{-}W_a = \rho_{H2O}V$. This permits computation of body volume.
  - ✓ Body density can be computed ρ_{body}=W/V.
  - ✓ Body fat can be computed from formulas.

Hydrostatic Bodyfat Testing in Air?

- Same methodology as Hydrostatic testing in water.
- What are the ramifications of using air?
  - ✓ Density of air is 1/1000th of water.
  - ✓ Temperature dependence of air.
  - ✓ Measurement of small volumes.
  - ✓ Used by NCAA Wrestling (there is a BodPod on PSU campus).

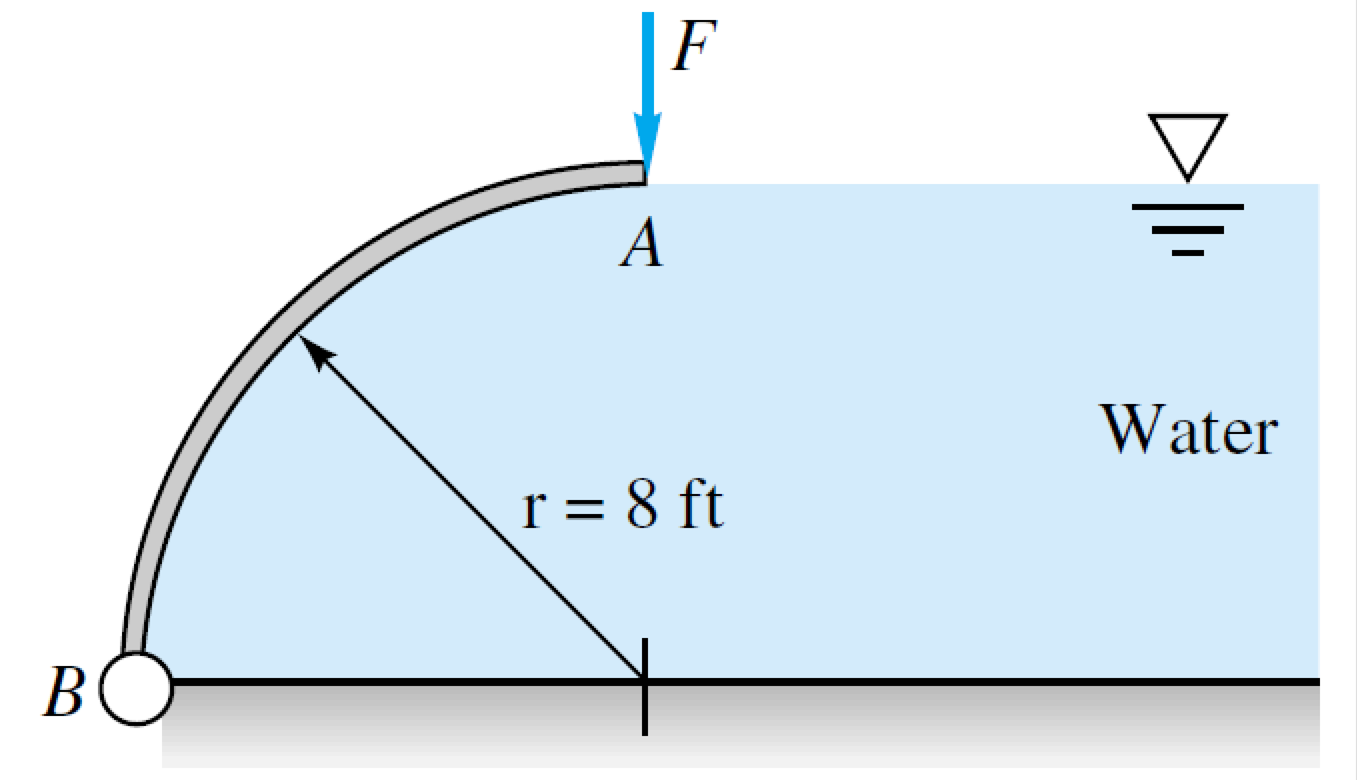
Exercices:

Gate AB in the Figure is a homogeneous mass of 180 kg, 1.2 m wide into the paper, hinged at A, and resting on a smooth bottom at B. All fluids are at 20°C. For what water depth h will the force at point B be zero?



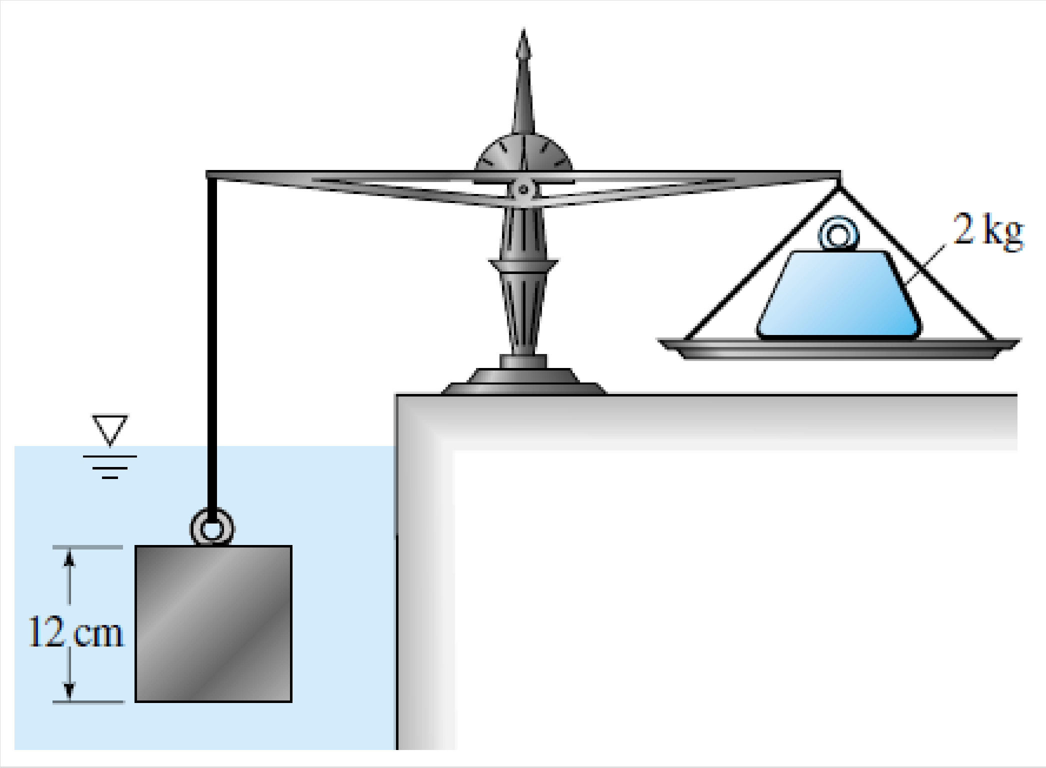
Exercices:

Gate *AB* in the Figure is a quarter circle 10 ft wide into the paper and hinged at *B*. Find the force *F* just sufficient to keep the gate from opening. The gate is uniform and weighs 3000 lbf.



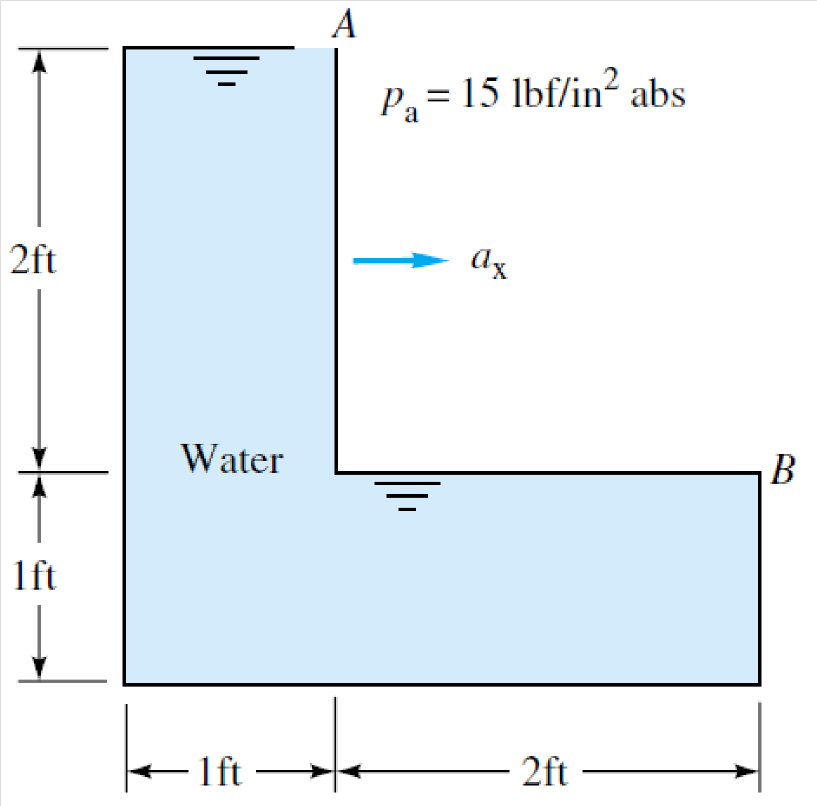
Exercices:

The homogeneous 12-cm cube in the figure is balanced by a 2-kg mass on the beam scale when the cube is immerssed in 20°C ethanol. What is the specific gravity of the cube?



Exercice:

The tank of water in the figure is full and open to the atmosphere at point A. For what acceleration a_x in ft/s^2 will the pressure at point *B be* atmospheric



Exercice:

The 45° V-tube in the figure contains water and is open at *A* and closed at *C*. What uniform rotation rate in rpm about axis *AB* will cause the pressure to be equal at points *B* and *C*? For this condition, at what point in leg *BC* will the pressure be a minimum?